2 The Structure of Physics

Physics describes the physical universe in terms of quantitative relationships between physical properties. It is a quantitative enterprise: relationships are expressed in the language of mathematics. Physical properties are quantifiable and therefore describable mathematically. The numerical values assigned to physical properties are often referred to as quantities. While quantifiable, properties are not themselves objects or processes, but concepts defined by the operations taken to assign them numbers (that is, to measure them). As long as a consistent means of assigning numerical values to a property exists, the property is useful to physics.

Only a (tiny) part of the universe can be analyzed at one time. To be analyzed, this part of the universe must be "detachable," somehow isolated, so that it can be studied independently of the rest of the universe. This isolated part of the universe can be called an "object," and the requirement of its isolation amounts to assuming that conditions can be established under which it behaves the same regardless of what happens in the rest of the universe.

A physical process is a sequence of interactions between objects. A collection of interacting objects is called a "system," a portion of the physical universe chosen for analysis. Everything outside the system is considered the "environment," which is ignored except for its effects on the system. Every model identifies a system and distinguishes it from its environment.

A systems theoretician might describe physics in terms of a uniform, general framework:

1. A system is a volume with a surface.
2. The system's state is characterized by the condition of the volume's contents.
3. Changes of state and other physical processs are described by accounting for transfers between the system and its environment as well as by what happens within the volume. (Supplementary information may be necessary to account for particular processes.)

From such a point of view, physics may be defined as that part of science which describes systems whose measured quantities are the values of the extensive properties mass, energy, momentum, angular momentum, electric charge, entropy, and amount of substance. Continuity and balance equations, conservation laws, and the law of entropy describe physical processes and changes of state.

A physical system is said to be "in a state" when all its physical properties have certain values. In practice, any physical system that can be investigated is influenced by a finite set of physical properties whose values range over restricted intervals. Too many influential or consequential properties whose values vary widely cannot be investigated in a controlled way.

To describe, or, better, model a physical process or a system, those physical properties, and only those physical properties, that contribute substantively to it must be identified. Physical properties whose values are constant during a process, or which do not change the values of those properties that do vary, can often be ignored. The "variables" measured when investigating a process or system are only those quantities whose variation influences the process or system.

Most, but not all, physical properties are measured spatially: at a point, on either side of a surface, or within a volume. For example, temperature, velocity, electrical potential, pressure, and density are measured at a point; the flow rate or magnitude of currents, such as electric, momentum, energy, and entropy currents, is a measure of the quantity of one of these properties that crosses a surface in time; and the amount of electric charge, mass, energy, momentum, and entropy refers to the quantity of these properties within a volume. A few properties, for example, length, electrical potential difference (voltage), and resistance are measured as differences between values at two points. Angles are obviously spatial measurements, but not at points, across surface, or in volumes. Time is temporal, not spatial, so measuring quantities associated with time, such as time interval, frequency, and period, requires a still different procedure. Finally, the quantity of volume does not itself *refer* to a spatial feature; it *is* a spatial feature: volume.

1. **Explain why length and electrical potential difference must be measured at two points.**

2. **How are time intervals and angles measured?**

2.1 Math, the language of science

Physical science is a quantitative enterprise: it's laws are expressed in the language of mathematics. Physical properties are defined quantitatively. That is, they are defined to be measured. Measurements, and the equations that describe, predict, or explain them, are quantitative in nature: they result in numbers, and therefore describable mathematically. Physics models tend to be constructed in mathematical terms or as computer algorithms.

The mathematics of physics assumes fluency in algebra. The following exercises are presumably elementary, but they should serve as a review and as a means for introducing the notation and terminology that will be used in this course.

A reference is available at
`http://prubin.physics.gmu.edu/courses/170/mathreview.pdf`

3. **If $ax^2 + bx + c = 0$, find x in terms of a, b, and c.**

4. **Expand and simplify:**

   (a) $(x+y)^2$
   (b) $(x-y)^2$
   (c) $(x+y)^3$

(d) $(x+y+z)^2$
(e) $(ax+b)(cx+d)$
(f) $(x+y)(x-y)$

5. Solve for t:

(a) $v_0t - y_0 = 0$
(b) $-\frac{1}{2}gt^2 + v_0t = 0$
(c) $-\frac{1}{2}gt^2 + v_0t + y_0 = 0$

6. Solve for a and T

$$m_1 g sin\theta - T = m_1 a$$
$$T - m_2 g = m_2 a$$

The predictive power of science is completely based on the universality and "objectivity" of mathematics. The result of mathematical calculations yield new symbols and structures that are then identified, by a leap of imagination, with physical properties, objects, and processes.

2.2 Extensive and intensive properties

Substance-current models often categorize physical properties as "extensive" or "intensive," according to how the quantity of the property changes when the size of the system changes.

The quantity of an extensive property is proportional to the size of the object or system. The quantity of an intensive property is independent of object or system size. Imagine dividing a homogeneous object in two: if the value of the property being measured remains the same, the property is intensive; if the value of the property is also divided, the property is extensive.

7. Is density an extensive or intensive property? Explain.

Quantities that refer to a volume, such as energy, momentum, electric charge, entropy, and amount of a substance, are likely to be extensive quantities and can be thought of (that is, modeled) as material entities, sometimes referred to as substance-like. In particular, they can be modeled as flowing from one volume to another, across surfaces, as a current or current intensity:

$$\text{current} = \frac{\text{amount of extensive quantity crossing a surface}}{\text{time interval}}$$

This is why the quantity 'volume' differs from other extensive quantities: it can't be conceived of as substantive or flowing between volumes.

If something is to flow, it must overcome the intrinsic resistance present in any system. It must be "driven." The driver in most cases is an intensive quantity or potential difference associated with the extensive quantity.

2.3 Dimensions and Units

Another name for physical properties is "dimensions." There are two types of dimensions, "base" and "derived." Combinations of base dimensions form derived dimensions. The mathematical relationship between properties symbolized in equations that describe real physical events must have the same dimension on each side of the (un)equal sign. If they don't, the mathermatical relationship is not physical. Values of different properties or dimensions cannot be added, but products (or ratios) of them can form useful derived dimensions.

Contemporary physics recognizes seven base dimensions. Those usually chosen are length, time, mass, electric current, temperature, amount of substance, and luminous intensity. It is important to note that this set of base dimensions is a matter of choice. What is required are the defining physical operations. Velocity, for example, can be thought of not as the ratio of two base dimensions (length and time), but rather in comparison to some reference, say, light speed, and can thus be chosen as a base dimension.

Two physical operations, comparison (that is, equality or inequality) and addition, specify a base dimension. Quantities with the same comparison and addition operations must be of the same base dimension, they must be values of the same physical property. The relation, $C = A + B$, implies that A, B, and C are all the same kind of property. No procedure exists for comparing or adding different properties or dimensions.

Comparison and addition, though physical, are reflected in mathematical operations with pure numbers: comparison obeys the transitive law of equality (if $A = B$, and $B = C$, then $A = C$) , while addition obeys the commutative law ($A + B = B + A$), the associative law $[A + (B + C) = (A + B) + C]$, and the identity law (if $A + B = C$, then, if $A + B + D = C$, $D = 0$) of addition.

Together, comparison and addition define, in completely physical terms, greater than and less than (if $B > 0$, and if $A+B = C$, then $C > A$), subtraction (if $A + B = C$, then $A \equiv C - B$), multiplication by a pure number (if $B = A + A + A$, then $B = 3A$), and division by a pure number (if $A = B + B + B$, then $B = A/3$).

Two pure numbers are compared either by finding the difference between them (subtraction) or by taking the ratio of one to the other (division). The difference between the pure numbers, A and B, then, is:

$$A - B \quad \text{or} \quad B - A$$

If $A - B$ is a positive number, then A is bigger (greater) than B:

$$A > B$$

If $A - B$ is a negative number, then A is smaller (less) than B:

$$A < B$$

If $A - B$ is zero, then A and B are equal:

$$A = B$$

For base dimensions, then, comparison and addition, as well as subtraction, multiplication by a pure number, and division by a pure number are all defined in physical terms. These operations are performed on dimensions of the same kind, yielding a dimension of that kind, perfectly analogous to pure number operations.

The converse of this statement is not true, that is, not all operations that can be performed on pure numbers–such as exponentials, logarithms, and transcendental operations–can be defined in physical terms. The ratio of two quantities of the same dimension is a dimensionless (pure) number (pay attention to units).

The ratio of A to B (both pure numbers) tells how many times B is (or how many Bs are) contained in A:

$$\frac{A}{B}$$

where A is denoted the numerator and B is denoted the denominator. A ratio of less than 1, which implies $A < B$, is often referred to as a fraction, which is the number that can be subtracted from A exactly B times, with no remainder.

8. **(a) How many times can 1/4 be subtracted from 1, leaving no remainder?**
   (b) From what number can 3/4 be subtracted four times without remainder?
   (c) What number can be subtracted from 29 seventy six times, leaving no remainder?
   (d) What number can be subtracted 14.4 times from 2.3 without remainder?

Ratios can (and often should) be written out as decimals.

The ratio of A to B (both physical properties with dimensions) may be interpreted as representing the amount of (whatever is being measured by) A associated with one chunk of (whatever is being measured by) B (the amount of A per one chunk of B).

9. A car uses V gallons of fuel traveling S miles.

   (a) How many gallons of fuel per mile did the car consume?
   (b) How many miles per gallon did the car get?
   (c) How many gallons of fuel would the car consume if it traveled $2S$ miles?
   (d) How far could the car go on $V/4$ gallons?

10. A rock that occupies a space (volume) of V m^3 (cubic meters) has a mass of m kg (kilograms).

(a) **Interpret the ratio m/V.**

(b) **Interpret the ratio V/m.**

(c) **What is the volume of a rock of the same material whose mass is $m/3$ kg?**

(d) **What is the mass of a rock of the same material whose volume is $1.5V$ m^3?**

11. **A car travels Δs miles in Δt hours.**

(a) **How far, on average, did the car travel in 1 hour?**

(b) **How many hours did it take the car to travel 1 mile?**

(c) **Moving at the same average rate, how far did the car travel in $0.75\Delta t$ hr?**

(d) **Moving at the same average rate, how long would it take the car to $6\Delta s$ miles?**

The defining operations of a base dimension, comparison and addition, permit the establishment of a standard, called a unit, such that quantity may be expressed in terms of multiples of the unit. Measurement amounts to adding replicas and fractions of the unit: the sum of these yields a numerical value of the property being measured. Notice that the physical addition of base properties of the same kind (it makes no sense to add base properties of different kinds), and the addition of numerical values based on the assignment of a unit, satisfy the same equation form, $A + B = C$, independent of the size of the unit. *The units on the left of the equal sign must be the same as the units on the right.*

The choice of unit is arbitrary. The property is physically the same, regardless of the unit, which leads to the conclusion that if a unit is scaled by a factor n, then the property's numerical value will scale by $1/n$. Therefore, *the ratio of two physical quantities of the same kind is independent of the unit size.*

Derived dimensions (physical properties) are defined in terms of some mathematical relationship between base dimensions. For example, a physical property derived from a monomial (a polynomial with only one term) relation between base dimensions takes the form of a power law:

$$D = \alpha A^a B^b C^c \ldots \tag{1}$$

where D is the quantity of a derived property, A, B, C, etc., are quantities of base properies, and the coefficient α and the exponents a, b, c, etc., are real numbers whose values define the derived property. The value of D, and of any derived property, depends on the choice of base units. But derived properties, like base properties, can be treated in terms of dimensions. Relationships between the base dimensions that comprise a derived quantity take the same form as the values of the properties in any choice of units.

If the value of some derived property D depends on the values of some number of length quantities, some number of mass quantities, some number of time quantities, etc., then the relationship might look something like

$$D = \alpha \ell_1^{\lambda_1} \ell_2^{\lambda_2} \dots m_1^{\mu_1} m_2^{\mu_2} \dots t_1^{\tau_1} t_2^{\tau_2} \dots$$

where the coefficient (or proportionality constant) α is a real number, as are the exponents, λ, μ, τ, etc. The dimension of D, then, could be written as

$$[D] = [L]^a [M]^b [T]^c [I]^d [\Theta]^e [N]^f [J]^g \tag{2}$$

where $[D]$ signifies "dimension of" D, $[L]$ dimension of length, $[M]$ dimension of mass, $[T]$ dimension of time, $[I]$ dimension of electric current, $[\Theta]$ dimension of temperature, $[N]$ dimension of quantity of substance, and $[J]$ dimension of luminous intensity.[1] In general, square brackets around a single property or quantity, or product of properites or quantities, indicate "dimension of."[2] Notice that terms of the same dimension are subsumed in one base dimension, and the powers are added: $a = \sum \lambda_i$, etc. Consider the dimension of density, ρ. Imagine a solid rectangular box whose mass m and sides, x, y, z, are all measured: $\rho = \frac{m}{V} = \frac{m}{xyz} = m^1 x^{-1} y^{-1} z^{-1} => [\rho] = [M]^1 [L]^{-3}$

The dimensions of many derived properties are, like density, the products of base dimensions to some power. Quantities of derived properties having the same dimension can, like those of base properties with the same dimension, be added. Quantities of derived properties can be multiplied and divided regardless of their dimensions, the result being another derived quantity with different dimensions. The ratio of quantities of derived properties whose dimensions and the powers to which they are raised are the same is a dimensionless (and therefore unitless) quantity, as in the case of a ratio of identical base properties. When the size of the unit of a base property is changed, all quantities of properties derived from that base property change by the same factor.

Special functions, including transcendental functions,[3] of dimensional derived properties are in general not derived properties because their values do not

[1] The symbols used to represent quantities and dimensions depend on context and often personal preference. The symbols most frequently used in this course are summarized in the following table:

Property	Dimension	Quantity
Length	L	ℓ, Δs, Δx $(y,\ z)$
Mass	M	m
Time	T	t, Δt
Electric Current	I	I_Q
Temperature	Θ	T
Amount of Substance	N	n
Luminous Intensity	J	I_v

[2] Common examples of this sort of formulation are velocity, $v = \frac{ds}{dt} = ds^1 dt^{-1} \Rightarrow [v] = [L]^1 [T]^{-1}$, and kinetic energy, $K = \frac{1}{2} m v^2 = \frac{1}{2} m \left(\frac{ds}{dt}\right)^2 = \frac{1}{2} m^1 ds^2 dt^{-2} \Rightarrow [K] = [m]^1 [v]^2 = [M]^1 [L]^2 [T]^{-2}$.

[3] Any (analytical, expressible as a power series) function that cannot be written as a polynomial is transcendental. Common examples are exponential, logarithmic, and trigonometric functions.

always transform like derived properties when base unit size changes. Only dimensionless arguments of these functions, which are unchanged by unit changes, leave the functions unchanged. Arguments of special functions are therefore dimensionless derived properties.

12. What are the base dimensions (and powers) of the following derived quantities:

(a) period [T]

(b) frequency [f] (1 / period)

(c) velocity [v]

(d) velocity squared [v^2]

(e) momentum [p] (mass × velocity)

(f) acceleration [a]

(g) momentum current [$I_{\vec{p}}$] (force [$\vec{F}$]) (mass × acceleration)

(h) work [W] [momentum current (force) × displacement in one dimension]

(i) gravitational potential energy [U_g] [mass × gravitational field strength (acceleration due to gravity) × height]

(j) kinetic energy [K] ($\frac{1}{2}$× mass × velocity squared)

(k) power [P] (energy / time)

(l) area [A]

(m) pressure [p] [momentum current (force) / area]

(n) volume [V]

(o) density [ρ] (mass / volume)

(p) charge [q] (electric current × time)

(q) electric field [E] [momentum current (force) / charge)

(r) voltage [V] (energy / charge)

(s) entropy [S] (energy / temperature)

(t) heat capacity (also thermal capacity) [C] [energy / (mass × temperature)]

13. Recalling that the arguments of special functions must be dimensionless, determine the dimensions of k ([k]) in each of the following expressions (Δx is a displacement, r is a radius):

(a) $cos(k\Delta x)$

(b) $e^{-k(\Delta x)^2/2r}$

(c) $2^{k/\Delta x}$

Analyzing dimensions and units can provide a low-level check of the correctness of a trial solution. Units and dimensions of all physical constants and properties must be consistent. They must be the same on both sides of an equal sign. For example, because

$$[M]^1[L]^1[T]^{-1} \neq [M]^1[L]^2[T]^{-2}$$

momentum can never equal kinetic energy, even if, by chance, the velocity happened to have a value of 2 in some units.

14. Analyze the dimensions on each side of the equal sign and determine if the following equations could be correct.

(a) $\frac{1}{2}\rho v^2 + \rho g h + p = E$, where ρ is density, v is velocity, g is the magnitude of the local gravitational field (acceleration due to gravity), h is altitude, p is pressure, and E is energy.

(b) $\Delta s = v_i \Delta t + \frac{1}{2}a(\Delta t)^2$, where Δs is displacement, v_i is the initial velocity, a is the constant acceleration, and Δt is the time interval.

(c) $m\Delta v = |I_{\vec{p}}|\Delta t = |\vec{F}|\Delta t$, where m is mass, Δv is the change of velocity, $I_{\vec{p}}$ is momentum current, F is force, and Δt is the time interval.

Furthermore, analyzing dimensions can often be used to guess or propose a formula or to estimate a solution. The key is to ensure that dimensions on each side of an equal sign are the same by determining the powers of the base properties that contribute to a derived property's dimension.

For example, in predicting a formula for the period, T, of a mass, m, oscillating at the end of a hanging spring with spring constant k, one might eliminate all properties not related to the mechanical behavior of an oscillating spring, that is, consider only length, mass, and time, or, in particular, the derived properties of period, acceleration due to gravity, and the spring constant, along with mass. The period has the dimension of time to the first power,

$$[T] = [T]^1.$$

This means that the properties in the formula must combine (raised to a power and multiplied) in a way that their derived dimension is also time to the first power, $[T]^1$. The dimension of the base property mass is

$$[m] = [M].$$

The property acceleration is derived from the base properties length (as displacement) and time (time interval), so

$$[a] = [L]^1[T]^{-2}.$$

Thus, the gravitational potential strength (acceleration due to gravity), g, has dimension,

$$[g] = [L]^1[T]^{-2}. \tag{3}$$

The dimension of the spring constant can be surmised from Hooke's law,

$$|I_{\vec{p}}| = |\vec{F}| = -k|\Delta\vec{s}|,$$

where k is the spring constant, and $\Delta\vec{s}$ is displacement, which has dimension $[L]$, and Newton's 2nd law,

$$I_{\vec{p}} = \vec{F} = m\vec{a},$$

where $\vec{a}$ is acceleration. The dimension of momentum current (force) is then derived from

$$[I_{\vec{p}}] = [\vec{F}] = [M]^1[L]^1[T]^{-2}$$

Putting all this together,

$$[k] = [M]^1[T]^{-2} \tag{4}$$

15. Derive the dimensional equation 4 from Hooke's and Newton's 2nd laws.

From all of this, an equation for the period of an oscillating spring can be surmised as follows:

1. Write a monomial expression including each potentially contributing quantity raised to a power:

   $$T = m^\mu k^\kappa g^\gamma$$

   where μ, κ, and γ are real numbers to be determined.

2. Substitute dimensions for each of the quantities and associate like dimensions:[4]

   $$[T] = [m]^\mu[k]^\kappa[g]^\gamma \Rightarrow$$

   $$\begin{aligned}[T]^1 &= ([M]^1)^\mu([M]^1[T]^{-2})^\kappa([L]^1[T]^{-2})^\gamma \\ &= [L]^\gamma[M]^{\mu+\kappa}[T]^{-2\kappa-2\gamma}\end{aligned}$$

   since $(b^c)^d = b^{cd}$.

[4] Recall that, because the product of two exponentials with the same base is the sum of the two exponentials' powers–$x^a \times x^b = x^{a+b}$–an exponential raised to a power is the product of the powers–$(x^a)^b = x^{ab}$.

3. Noting that on the left side of the equation,[5] $[L]^0[M]^0[T]^1$, equate the exponents on the two sides of the equation:

$$\begin{aligned} 0 &= \gamma \\ 0 &= \mu + \kappa \\ 1 &= -2(\kappa + \gamma) \end{aligned}$$

and solve for each exponent:

$$\begin{aligned} \gamma &= 0 \\ \kappa &= -\frac{1}{2} \\ \mu &= \frac{1}{2} \end{aligned}$$

leading to the dimensional equation,

$$[T] = [m]^{1/2}[k]^{-1/2}[g]^0,$$

and a reasonable guess for the equation of a mass oscillating at the end of spring:

$$\begin{aligned} T &\propto \sqrt{\frac{m}{k}} \Rightarrow \\ T &= \alpha\sqrt{\frac{m}{k}} \end{aligned}$$

where α is a dimensionless constant to be determined by experiment.

16. Which is the correct formula for the period of a pendulum of length ℓ, $T = 2\pi\sqrt{\ell/g}$ or $T = 2\pi\sqrt{g/\ell}$? Justify your answer by analyzing dimensions.

17. What are the dimensions of a and b in the equation

$$E = a\ell sin(bt)$$

where E is energy ($[E] = [M]^1[L]^2[T]^{-2}$), ℓ is a length, and t is time.

18. In the equation

$$\Delta E = mC\Delta T$$

[5] Any value other than 0 raised to the power 0 equals 1.

where $\boldsymbol{\Delta E}$ is a quantity of energy transferred to or from a thermodynamic system, $\boldsymbol{m}$ is the mass of the system, $\boldsymbol{\Delta T}$ is the temperature change, and $\boldsymbol{C}$ is the "heat capacitance." From which base dimensions is heat capacitance derived and how are they combined?

2.4 Continuity or transport equations

The quantity of an extensive property X contained in a volume satisfies a continuity or transport equation[6]

$$\frac{\Delta X}{\Delta t} \equiv I_X + \sigma_X \tag{5}$$

where[7]

- $\frac{\Delta X}{\Delta t}$ represents the rate at which the quantity of the property X within the volume changes with time t (if $\frac{\Delta X}{\Delta t} > 0$, the quantity within the volume is increasing; if $\frac{\Delta X}{\Delta t} < 0$, the quantity within the volume is decreasing);
- I_X refers to the flow or current of X through the surface of the volume (if $I_X > 0$, X is flowing into the volume; if $I_X < 0$, X is flowing out of the volume); and
- σ_X is the rate at which X is created ($\sigma_X > 0$, referred to as a source) or destroyed ($\sigma_X < 0$, referred to as a sink) within the volume.

Equation 5 thus states that the rate at which the quantity of property X changes depends on how much of X crosses the surface of the volume (that is, the magnitude of a current or flow into or out of the volume), and on how much of X is created or destroyed within the volume.

For some extensive properties, such as energy, momentum, and electric charge, $\sigma_X = 0$ always. None of these properties can be created or destroyed. They are **conserved**, and their quantity inside a volume changes only due current flow. Entropy, though, can be created, but not destroyed. Thus, $\sigma_S \geq 0$ (S is the symbol for entropy). In the absence of outward current flow, entropy can only increase or remain the same within a volume. Amount of a substance

[6] The upper-case Greek letter Δ ("delta") has many meanings in mathematics and elsewhere. In this course, it is used in two ways:

1. The (macroscopic) amount of change in a quantity:

   $$\Delta X \equiv X_2 - X_1$$

   The lower-case Latin letter d is used in calculus (derivatives and differential) for infinitesimal changes in a quantity.

2. A small portion of the quantity of a property (see below)

[7] The $\equiv$ symbol is read as "is defined by" or "is identical with" or "is equivalent to."

within a volume can increase or decrease without current flow. Neither entropy nor amount of substance is conserved.

An assertion that an extensive physical property is or is not conserved has often been regarded as among the most important, if not the most important, of physical laws. Some have gained sufficient status, typically following contentious debate, to be assigned a name. For example,

- Newton's second law: Momentum cannot be produced or destroyed.
- The first law of thermodynamics: Energy cannot be produced or destroyed.
- The second law of thermodynamics (law of entropy): Entropy can be produced, but not destroyed.

Debates about these and other assertions frequently centered on whether or not the physical property that is the focus of the assertion is extensive or intensive.

Conservation of electric charge has no special name, perhaps because the extensivity of charge was accepted before the conservation law was settled. Similarly with amount of substance, whose extensivity and non-conservation were obvious.

For any conserved extensive property, X, $\sigma_X = 0$. Equation 5 therefore defines the flow (current) of that extensive quantity as the rate at which it changes within the volume of interest:

$$I_X \equiv \frac{\Delta X}{\Delta t} \tag{6}$$

when X is a conserved extensive property.

If $X \equiv Q$, for example, where Q is the symbol for electric charge, then, because electric charge is conserved, $\sigma_Q = 0$, and Equation 5 becomes the definition of electric current:

$$I_Q \equiv \frac{\Delta Q}{\Delta t} \tag{7}$$

If $X \equiv \vec{p}$,[8] where p is the symbol for momentum, then, because momentum is conserved, $\sigma_{\vec{p}} = 0$, and Equation 5 becomes the definition of momentum current, more familiarly called "force," $\vec{F}$:

$$I_{\vec{p}} \equiv \frac{\Delta \vec{p}}{\Delta t} \equiv \vec{F} \tag{8}$$

Note that this is a form of Newton's second law.

Energy E is distributed in space and can flow. If $X \equiv E$, then, because energy is conserved, $\sigma_E = 0$, and Equation 5 becomes the definition of energy current, commonly referred to as "power," P:

[8] $\vec{a}$ indicates that the property a is quantified as a vector, having both magnitude and direction, in contrast to a scalar property, quantified with simply magnitude. Thus, quantifying momentum, $\vec{p}$, and momentum current, $I_{\vec{p}}$ (force, $\vec{F}$), requires both numbers and directions (angles), while quantifying charge, Q, energy, E, and power, P, requires just numbers.

$$I_E \equiv \frac{dE}{dt} \equiv P \tag{9}$$

In the three examples just presented, the terms I_Q, $I_{\vec{p}}$ (or $\vec{F}$), and I_E (or P), represent the strength [and, in the case of $I_{\vec{p}}$ ($\vec{F}$), direction] of currents, that is, how rapidly the substance-like (extensive) properties Q, p, and E, are changing (increasing if the current is greater than zero or decreasing if it is less than zero) within the volume of under investigation (the system).

An extensive property and its associated current can characterize every major subdivision of classical physics: momentum and momentum currents characterize mechanics; electric charge and electric currents characterize electromagnetism; entropy and entropy currents characterize thermodynamics; and the amount of substance and matter currents characterize chemistry.

Energy and energy current (power) apply universally to all of physics.

2.5 "Forms" of Energy

Modern physics is modeled as particles, interactions, fields, and waves. The Special Theory of Relativity has rendered superfluous the notion that energy appears in various forms or guises, such as electric, chemical, free, nuclear, thermal, rest, and radiant energy, as well as kinetic, potential. The modern view is that all "forms" of energy are fundamentally associated with motion (kinetic energy) or relative position (potential energy).

Under the substance-current model, energy flows, carried with the current of another extensive quantity. Under this model, it is sufficient to specify just the other extensive quantity.

2.6 Energy currents

Again, according to the substance-current model, energy is transferred by a carrier of energy. An energy current (energy flow rate, power), P, always accompanies and is proportional to one or more extensive-property currents:

$$P \propto I_X,$$

where $\propto$ means "proportional to", I is a current, and X is an extensive property. For example, the mechanical energy current, $P \propto |I_{\vec{p}}|$ ($|\vec{F}|$), where $|I_{\vec{p}}|$ ($|\vec{F}|$) indicates the magnitude of the vector $I_{\vec{p}}$ ($\vec{F}$), momentum current (force). For electricity, $P \propto I_Q$, electric current, often written without the subscript, I. For thermodynamics, $P \propto I_S$, entropy current. Whenever there is an entropy, momentum, electric charge, and/or some other extensive property current, there is always an energy current. Thus, extensive properties, such as entropy (S), momentum ($\vec{p}$), electric charge (Q), and amount of substance (n), can be seen to act as energy carriers.

The proportionality term in each of these relations is the so-called "energy-conjugated intensive property," an intensive property associated with the extensive property that carries the energy. In fact, these relations define the associated energy-conjugated intensive properties.

The electrical potential (or voltage) V is defined by

$$P = \Delta V_Q I_Q \tag{10}$$

where ΔV_Q is electrical potential difference.

Velocity is defined by[9]

$$P = \vec{v} \cdot I_{\vec{p}} = \vec{v} \cdot \vec{F} \tag{11}$$

Energy flows also with thermal and chemical currents. For thermal transport, the relationship $P \propto I_S$ defines the intensive property "temperature," T

$$P = \Delta T I_S \tag{12}$$

where S signifies entropy and ΔT is temperature difference.

For chemical transport, the relationship $P \propto I_n$ defines the intensive property "chemical potential," μ

$$P = \Delta \mu I_n \tag{13}$$

where n indicates amount of substance and $\Delta\mu$ is chemical potential difference.

These formulas, known as "balance equations," express that:

- whenever there is an electric current, there is also an energy current whose magnitude depends on the strength of the electric current and the magnitude of the electrical potential difference;
- whenever there is a momentum current (force), there is also an energy current whose magnitude depends on the strength of the momentum current (force) and the magnitude of the velocity;
- whenever there is an entropy current, there is also an energy current whose magnitude depends on the strength of the entropy current and the magnitude of the temperature difference;

and so forth. In short: extensive property currents carry an energy current whose magnitude depends on the strength of the extensive property current and the quantity of the associated energy-conjugated intensive property.

Different extensive properties may be coupled to one another more or less strongly. For example, mass and amount of substance are very strongly coupled,

[9] $\cdot$ is the symbol for a specific sort of multiplication (called a dot or scalar product) between vectors that results in a scalar (a term that has no directional information, only magnitude). It has the value, $\vec{a} \cdot \vec{b} \equiv |\vec{a}||\vec{b}| \cos\theta$, the product of the magnitudes of the vectors times the cosine of the angle, θ, between the two vectors $\vec{a}$ and $\vec{b}$.

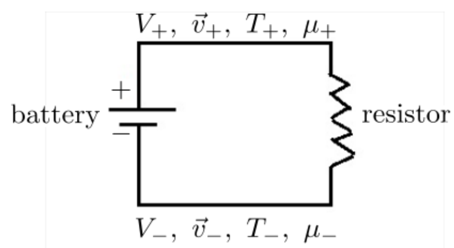
while entropy and mass are very weakly coupled, though sufficiently coupled such that, for example, radiators function. Therefore, it should be expected that multiple extensive property currents contribute to a given energy current. In principle, in fact, they all always do,

$$P = \Delta V_Q I_Q + \vec{v} \cdot \vec{F} + \Delta T I_S + \Delta\mu I_n + \cdots \tag{14}$$

although the contribution in any given instance from most of these terms is negligible, due either to a barely existent current or an extremely small quantity of the conjugate intensive property.

Imagine a metal bar arranged so that one end is over a Bunsen burner while the other end is embedded in a block of ice. The temperature of the two ends–and every point in between–will be different. Consequently, entropy will flow, and therefore, since $\Delta T I_S \neq 0$, so will energy. Yet, no momentum or electric charge will flow, and while quasiparticles known as phonons will flow, the chemical potential difference will be zero. Thus, the only energy carrier in this situation is entropy.

Electric currents flow in closed loops (called "circuits"), from a source of potential difference, through conductors and resistors, and back to the source. No charge accumulates along a closed circuit, not even in the source. Only as much charge leaves the source as comes in. No charge enters or leaves the circuit. A battery is not a source of charge, but rather a pump that releases energy and causes charge to flow. Besides batteries, generators, solar cells, and thermocouples are also sources. Batteries get their energy from chemical potential, angular momentum carries energy to generators, light carries energy to solar cells, and entropy carries energy to thermocouples.



Consider an electric circuit with a potential source (here, a battery) and a resistor [see Figure]. Charged particles flow through the wires. These charged particles have charge, mass, momentum, entropy, and amount of substance, and so, carrying the energy current are electric, mass, momentum, entropy, and substance currents. However, the mass of these particles is very small. So is their velocity in an electric circuit through wires. Their momentum is therefore negligible, leaving electric, entropy, and substance currents to consider.

In a closed circuit, there are both outbound and return paths, from and to the potential source (by convention, from the positive terminal of a battery to the negative terminal), leading to two sets of quantities for the intensive properties:

Outbound: V_+, T_+, μ_+
Return: V_-, T_-, μ_-

where the subscript + refers to the outbound path, and subscript − refers to the return path. The currents, I_Q, I_S, and I_n, are continuous; there is never evidence of a quantity of any of these extensive properties accumulating anywhere along the circuit.

The power consumed (rate of energy released to the environment) by the device (whatever the resistance does–give off light, get hot, etc.) is the difference between two sums of current-intensive property products:

$$P = (V_+ - V_-)I_Q + (T_+ - T_-)I_S + (\mu_+ - \mu_-)I_n$$

Experiment, however, shows that, in general, $T_+ \approx T_-$ and $\mu_+ \approx \mu_-$, so all that remains is,

$$P = (V_+ - V_-)I_Q = \Delta V_Q I_Q$$

Energy is carried by electric current, driven by electric potential difference, and the rate at which the energy flows (power) is the product of these two quantities.

Consider a small portion of water, Δm,[10] falling from height h_2 to height h_1. By falling, the water releases (potential) energy

$$\Delta E = \Delta mg(h_2 - h_1) \tag{15}$$

If the mass increment Δm traverses the distance $h_2 - h_1$ in time interval Δt, then a mass current

$$I_m = \frac{\Delta m}{\Delta t} \tag{16}$$

flows from h_2 to h_1. This mass current carries the energy current

$$P = I_E \equiv \frac{\Delta E}{\Delta t} \tag{17}$$

Substitution leads to

$$P = g(h_2 - h_1)I_m \tag{18}$$

Identifying the "gravitational potential," V_g[11]

$$V_g \equiv gh \Rightarrow \Delta V_g \equiv g\Delta h \tag{19}$$

the energy current (power) can be written

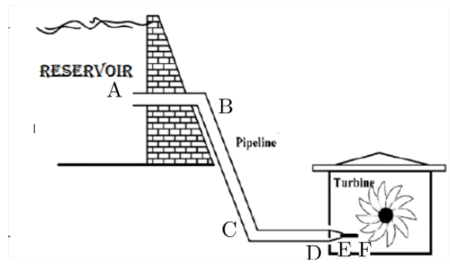
$$P = (V_{g2} - V_{g1})I_m = \Delta V_g I_m \tag{20}$$

analogous with the other energy current equations: a gravitational potential difference drives a mass current which carries energy.

[10]The symbol Δ here means "increment" (a finite quantity, by which a variable quantity is increased or decreased), rather than "difference" or "change." The whole is the sum of all increments.

[11]As with most potentials, the physically significant quantity is the difference at different positions, not the values at a single point. The reference point–the zero potential–is an arbitrary choice, but must be consistently applied to the entire volume.

19. Derive Equation 20 from Equations 15 - 19.



Falling water, of course, can be put to practical use. Consider a hydroelectric power plant [see Figure]. Since water is uncharged (neutral) and the temperature of the water changes very little with height, entropy and electric current play no part in the process. The extensive quantity currents remaining to consider are mass, amount of substance, and momentum, I_m, I_n, and $I_{\vec{p}} = \vec{F}$. The associated potentials are those of the conjugate intensive properties, V_g, μ, and $\vec{v}$.

For the following analysis, the zero point of the gravitational potential is chosen to be at the level of the bottom of the turbine. The nozzle at the base of the pipeline greatly restricts the water flow between A and E, so the magnitude of the water velocity between points A and E of the pipeline is very small and changes very little until the water exits the nozzle. Except under very extreme conditions, water is incompressible nor does it disperse. Under most circumstances, its density changes very little. Taken together, this implies that the mass flow in the pipeline is constant.

The section of the pipeline between A and B slopes very little, so the pressure difference between points A and B is small. Because chemical potential is proportional to pressure,[12] chemical potential at the two points, and the difference between them will be small, too: $\mu_\text{A} \approx \mu_\text{B}$. The only carrier of energy in this section of the pipeline, then, is mass, and the driver is gravitational potential:

[12]Recall that, for a falling portion of mass,

$$\Delta E = \Delta mg\Delta h = \Delta V_g \Delta m$$

where $V_g \equiv gh$ is the gravitational potential. But energy can also be carried by amount of substance

$$\Delta E = \Delta\mu\Delta n.$$

For continuous energy flow in which the carriers happen to change from gravitational to chemical potential, these two expressions must be equal,

$$\Delta V_g \Delta m = \Delta\mu\Delta n.$$

so

$$\Delta\mu = \Delta V_g \frac{\Delta m}{\Delta n}.$$

Gravitational pressure is defined as the density,

$$\rho \equiv \frac{m}{V},$$

where V is the volume, times the gravitational potential V_g.

$$\Delta p \equiv \Delta\rho\Delta V_g.$$

Thus,

$$P_{\mathrm{BA}} = (V_{g_{\mathrm{B}}} - V_{g_{\mathrm{A}}})I_m + (\mu_{\mathrm{B}} - \mu_{\mathrm{A}})I_n = \Delta V_{g_{\mathrm{BA}}} I_m$$

Between B and C, ΔV_g is large, since Δh is large. Also, the water pressure increases between B and C, so, both gravitation and chemical potential differences are drivers:

$$P_{\mathrm{CB}} = (V_{g\mathrm{C}} - V_{g\mathrm{B}})I_m + (\mu_{\mathrm{C}} - \mu_{\mathrm{B}})I_n = \Delta V_{g_{\mathrm{CB}}} I_m + \Delta\mu_{\mathrm{CB}} I_n$$

and mass and amount of substance carry the energy in this section of the pipeline

The physical situation between C and D is essentially the same as that between A and B,

$$P_{\mathrm{DC}} = \Delta V_{g_{\mathrm{DC}}} I_m$$

and gravitational potential drives the current while mass carries the energy.

From one side of the nozzle, at D, to the other side, at E, water exits the nozzle as the pressure drops dramatically to that of the atmosphere. Here, chemical potential difference drives the current and amount of substance carries the energy.

$$P_{\mathrm{ED}} = (\mu_{\mathrm{E}} - \mu_{\mathrm{D}})I_n = \Delta\mu_{\mathrm{ED}} I_n$$

The total magnitude of the energy current, that is, the total power released from the nozzle is,

$$\begin{aligned} P_{\mathrm{EA}} &= P_{\mathrm{BA}} + P_{\mathrm{CB}} + P_{\mathrm{DC}} + P_{\mathrm{ED}} \\ &= \Delta V_{g_{\mathrm{BA}}} I_m + \Delta V_{g_{\mathrm{CB}}} I_m + \Delta\mu_{\mathrm{CB}} I_n + \Delta V_{g_{\mathrm{DC}}} I_m + \Delta\mu_{\mathrm{ED}} I_n \\ &= \Delta V_{g_{\mathrm{DA}}} I_m + \Delta\mu_{\mathrm{EA}} I_n \\ &= \Delta V_{g_{\mathrm{EA}}} I_m + \Delta\mu_{\mathrm{EA}} I_n \end{aligned} \tag{21}$$

Since $V_{g_{\mathrm{E}}} \approx V_{g_{\mathrm{D}}}$, $\mu_{\mathrm{B}} \approx \mu_{\mathrm{A}}$, and $\mu_{\mathrm{D}} \approx \mu_{\mathrm{C}}$. If A is near the surface of the reservior and far above the nozzle, then $\mu_{\mathrm{E}} \approx \mu_{\mathrm{A}}$, so $P_{\mathrm{EA}} \approx \Delta V_{g_{\mathrm{EA}}} I_m$, as would be expected of free fall.

20. Verify Equation 21 by expanding each potential difference and carrying out the algebra.

The velocity of the water out of the nozzle increases as dramatically as the pressure drops, and the energy that was carried with the water is now carried by the momentum of the water, driven by the velocity:

$$\Delta\mu = \frac{\Delta p}{\Delta\rho}\frac{\Delta m}{\Delta n} = \Delta p\frac{\Delta V}{\Delta n}$$

after substituting the definition of density and simplifying. As can be seen, the chemical potential, at least for liquids, is proportional to the pressure, and the proportionality constant, $\frac{\Delta V}{\Delta n}$, is known as the "molar volume." See, also, the thermodynamics unit.

$$P_{\mathrm{FE}} = \vec{v}_{\mathrm{F}} \cdot I_{\vec{p}} = \vec{v}_{\mathrm{F}} \cdot \vec{F} = \Delta V_{g_{\mathrm{EA}}} I_m + \Delta \mu_{\mathrm{EA}} I_n$$

assuming no losses. This is the power that drives the turbine which transforms it into electricity.

21. The reservior of a hydroelectric power plant sits at height H above the turbine, which it supplies with water at a rate of $\frac{\Delta V}{\Delta t}$ (given in terms of volume of water per unit time interval; the density of water at the temperature of the reservoir is $\rho_{H_2 0}$). What is the energy current (power) produced by the power plant (assuming no losses and $\mu_H = \mu_0$)?

Notice how the carrier changes over the course of the energy flow. This is not unique to hydroelectric plants, but rather is a common feature of many systems.

Also take note that, as with an electrical cicruit, the hydroelectric plant, and nearly all circuits, contain some resistance to current flow–a sort of friction. This resistance is overcome by a driving interaction. While each extensive physical property can flow as a current and carry energy, for the extensive property and energy to flow, there must be a driver. A driver may be internal or external to the system.

Wherever there is resistance to flow, entropy is created and can flow out of the system, carrying energy along. This is known as a loss, but it's just a transfer of energy from the system to the environment.

Extensive property current tends to flow from a position where the quantity of the conjugate intensive property is large toward a position where the quantity of the conjugate intensive property is small, carrying energy along:

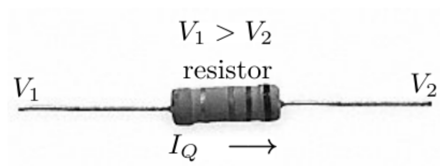
$$\begin{aligned}
P &= (T_2 - T_1) I_S = \Delta T I_S \\
P &= (\vec{v}_2 - \vec{v}_1) \cdot I_{\vec{p}} = \Delta \vec{v} \cdot I_{\vec{p}} = (\vec{v}_2 - \vec{v}_1) \cdot \vec{F} = \Delta \vec{v} \cdot \vec{F} \\
P &= (V_2 - V_1) I_Q = \Delta V_Q I_Q
\end{aligned}$$

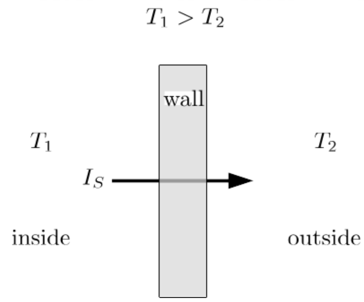
Sufficiently large positional differences between quantities of a conjugate intensive property can overcome resistance to the associated extensive property current, which then flows spontaneously, carrying energy–no external driver is required. An external driver is required to overcome greater resistance or to cause an extensive property to flow against the difference between intensive property quantities.

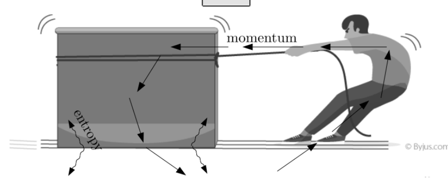
When electric charge flows through an electric conductor that is not perfect [see Figure], the current is hampered by the resistance of the conductor, but flows due to an electric potential difference created by a battery or some other "electricity pump."

In Winter, entropy flows to the outside through the walls of buildings. A wall is a thermal resistor, due to an air gap and possibly insulation, so entropy currents encounter resistance. Entropy flows because a temperature difference drives it (it's warmer inside than outside). A temperature difference can be generated in two ways: through the creation of entropy or by means of a heat pump.

A momentum current flows between objects that slide relative to one another along a common suface interface. The sliding is resisted by friction, which the momentum current has to overcome. Momentum will flow from the faster object to the slower one only if the objects' velocities differ. Velocity difference is the driver of momentum currents, and velocity differences are generated by an engine, for example. These results are summarized in the following table.





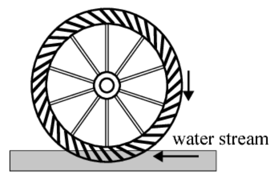


extensive quantity	current	resistance	driver	mechanism
electric charge Q	electric I_Q	electric R	electric potential difference $\Delta V_Q \neq 0$	battery, generator, solar cell
entropy S	entropy I_S	thermal	temperature difference $\Delta T \neq 0$	heat pump
momentum $\vec{p}$	momentum $\vec{F}$ (force)	friction, momentum	velocity difference $\Delta \vec{v} \neq 0$	motor, engine

22.

Which extensive quantities flow together with the water that drives a water mill? Which of these quantities is responsible for the energy transfer to the water wheel? Why do the other quantities not play a role?

23.



An undershot water wheel (a vertically-mounted wheel rotated when water strikes paddles or blades at the bottom of the wheel) discharges energy from flowing water by changing its velocity. What is the pertinent energy carrier?

24. For a compressed air machine only the chemical potential term $(\mu_1-\mu_2)I_n$ is relevant to the energy exchange. Why?

25. A pump presses a volume ΔV per unit time interval Δt into a water supply network with an excess pressure Δp. How much energy per unit time interval, P, does the pump consume (See footnote 12 for the relationship between chemical potential, pressure, and volume, and recall that $I_n \equiv \frac{\Delta n}{\Delta t}$.)

Notice how the overarching model, in this case the substance-current model, is applicable to the subfields of classical physics: different subfields share relationships of the same form, such that the subfields are related by analogy, simply by substituting appropriate quantities. Notice, too, that energy and its flow (power), as well as position and time, transcend subfield.