Motion: Week 1

Sections 4.1 – 4.8, pages 1 - 15

- Rectilinear (one-dimensional) motion
- Position and displacement [or distance or length]
- Instant (clock reading) and time interval
- Corresponding displacements and time intervals
  - Ordered t-s pairs
- Position versus time graph
- Average velocity
- Uniform (constant) motion
  - Position versus time graph of uniform motion
  - Equations of straight lines
- Nonuniform motion
- Slopes, derivatives, and instantaneous velocity
- Velocity versus time graph

Introduction

- Motion is the fundamental characteristic of all physical processes
- Motion is described in terms of two quantities that transcend sub-fields of physics: position and time

- To some extent, physics is the study of motion
- Foundational principles of all modern physics theories include Newton's three laws of motion (2, 4, and 5 in list of modern theory characteristics)
- Here, the goal is to describe motion, rather than understand when and how it changes

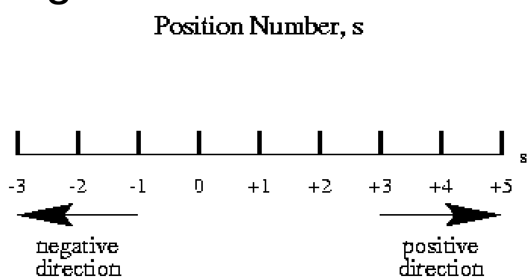
Rectilinear Motion

- Motion (in either direction) along a straight line (one-dimensional)

- Reality, in all its complexity, is not the primary object of physics inquiry; isolating—often by abstraction—interesting aspects of reality in order to understand them is
- The starting point is usually a simple abstraction: the study of motion starts with one-dimensional (rectilinear) motion

Position

- Position: A numerical value along a coordinate scale/axis (or one-dimensional reference frame) with an arbitrarily chosen origin and orientation
  - Origin denoted 0
  - Values assigned in ascending magnitude in either direction from 0
  - Spacing between values (units) is also arbitrary, but best if a standard is followed



- Position: One of the two basic concepts in the description of motion (and thus of nearly all physics)
- Position, in one or more dimensions, is normally quantified by reference to a series of numerical markers, whose zero, orientation, and spacing are all arbitrary choices made for convenience of assigning and communicating these numbers: a reference frame is comprised of one or more coordinate scales or axes
- Between markers: fractional values

Position

- Generic symbol for rectilinear position number: s
  - Dimension: $[L]^1$
  - For individual positions, use s with subscripts
  - Subscripts distinguish individual position readings and associated clock readings (instants)
- For rectilinear motion, right is positive, left negative
- For multi-dimensional space, use multiple—usually perpendicular—axes or frames (eg., x, y, z)

- Mathematical descriptions require symbolic representations of quantities
- s is position, not displacement (distance), which is the difference between two positions
- <u>*Directional convention employed throughout the course: increasing, positive, right; decreasing negative, left*</u>

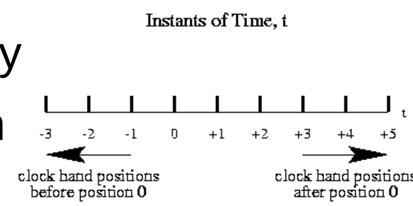
Displacement (Distance)

- The arithmetic difference between two position numbers: $\boldsymbol{\Delta s \equiv s_2 - s_1}$
  - May be positive, negative, or zero
  - Dimension: $[\boldsymbol{L}]^{\mathbf{1}}$
- Motion: displacement at different instants (clock readings)

- The algebraic properties of displacement (positive, negative, zero) indicate direction
  - If the later position is bigger (more positive, to the right) than the earlier position, the displacement is positive (to the right); if it is smaller (more negative—sign is important; it's not just magnitude), displacement is negative (to the left); if they're the same, displacement is zero
  - Bigger = more positive; smaller = more negative
- Problems 1-5: practice the arithmetic of position and displacement
- <u>*Understand the mathematical notes. The notions of increasing and decreasing, positive and negative directions, and magnitude are fundamental to the course*</u>

Instants and Intervals of Time

- Instant ≡ clock reading, *t*
  - $[t] = [T]^1$
- Time interval: the arithmetic difference between a later and an earlier instant, $\Delta t \equiv t_2 - t_1 \;\; ([\Delta t] = [T]^1)$
  - $t_2 > t_1$
  - Can be positive or zero, but not negative
- Assignment of time origin, $t_0 = 0$, arbitrary
  - Instants can be negative, if they happen before t_0, but intervals cannot be



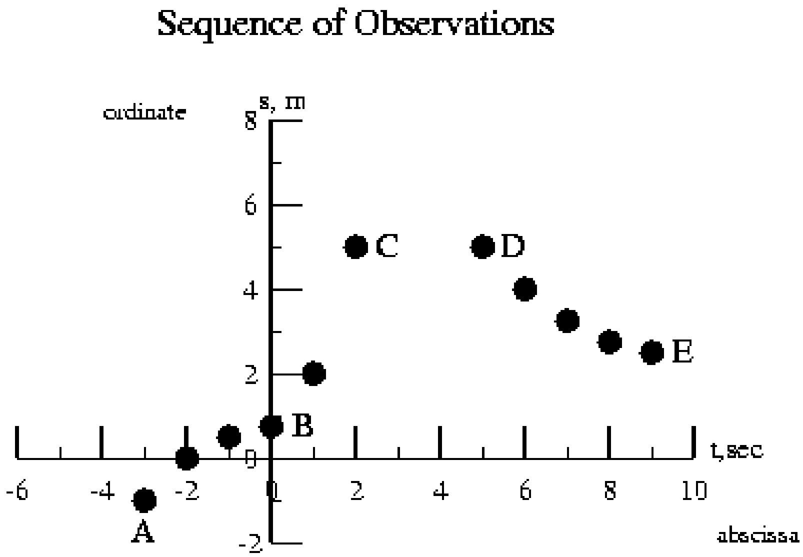
- Instant (clock reading): the other basic concept in the description of motion (and thus of nearly all physics)
- Time measurement also involves some arbitrary choices —primarily the time origin and, to a lesser extent (because there are fewer standard units), spacing
- Measuring intervals of time: counting a regular variation or oscillation, so sequencing numbers is possible, always increasing (more positive)—time flows forward
- Distinct interpretations for subtraction order: later – earlier = how much time has passed [time interval]; earlier – later = how much time before did the earlier event occur [not (strictly) a time interval]—Delta t > 0

Displacement and Corresponding Time Interval

- Associating clock readings and position measurements, and thus time intervals and displacements, can indicate motion
- Corresponding displacements and time intervals ⇒ corresponding positions and instants (clock readings)
  - Ordered pairs: (t_j, s_j)

- Motion for sure: Neither Delta s nor Delta t is equal to zero
  - Ambiguous: Delta s = 0 when Delta t > 0
- Note the implicit assumption that the reference coordinate axis is fixed, but this need not be the case (relativity)

Position vs Time Graphs of Rectilinear Motion



- Ordered pairs plotted on a 2-dimensional graph (scatter plot)
  - A vs B means A is plotted along the ordinate (vertical axis) and B is plotted along the abscissa (horizontal axis)
- Note that what is illustrated is not the path taken by the object; this is one-dimensional motion: one can infer/guess the motion by interpreting the plot
- Problems 6-7 practice such interpretations

The Number $\frac{\Delta s}{\Delta t}$ (Average Velocity)

- Ratio of displacement to corresponding time interval suggests rate of motion
  - If monotonic, magnitude implies rate (per unit time) of motion
  - Direction given by displacement ($\Delta t > 0$ always)
  - For more general motion, magnitude implies the average rate of a displacement from initial to final positions
  - Too coarse a number to distinguish uniform from non-uniform motion

- Concepts are defined by how numbers are assigned to quantities
- New concepts, particularly when derived from old concepts, require interpretation
- Interpretation involves determining the conditions under which the numbers take the values they do
- This ratio again involves corresponding displacements and time intervals
- Problem 8: determine which of the 5 characteristics applies to each graph in problem 7 and say what happens to the average velocity in each case

Average Velocity, $\bar{v}$

- $\bar{v} \equiv \frac{\Delta s}{\Delta t}$
- Derived quantity: $[\bar{v}] = [L]^1[T]^{-1}$
- Ratio of average velocities (eg., $\frac{\bar{v}_A}{\bar{v}_B}$) is a pure number indicating the numerator ($\bar{v}_A$) is “so many times” the denominator ($\bar{v}_B$)

- There are many sorts of means, but average implies arithmetic mean (sum / number of values); seeing how this works here requires integral calculus
- Problem 9: further thinking about the ratio—both its value and its meaning

Uniform Motion

- Equal displacements in equal time intervals
- Average velocity is the same for all time intervals and corresponding displacements
- For uniform (constant) motion <u>only</u>, the bar is unnecessary: $\bar{v} \to v$

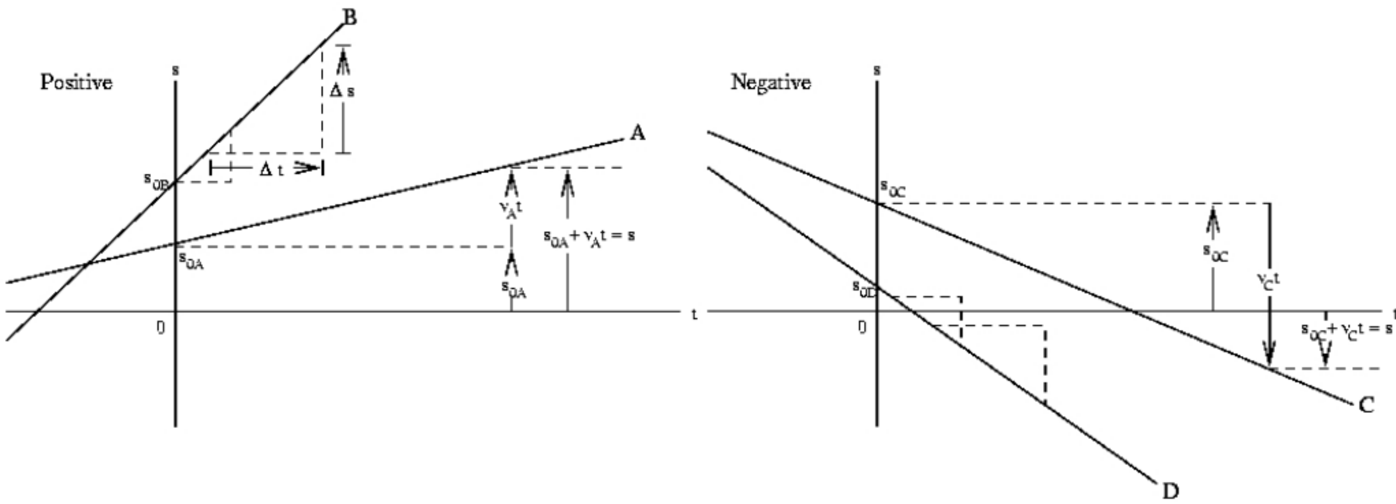
- For uniform (constant) motion, instantaneous and average velocity are the same

Position vs Time Graph for Uniform Motion

- Label earlier clock reading t_i, and later clock reading t_f, or, more generally t
  - Corresponding position measurements: s_i, s_f, s
- $$v \equiv \frac{\Delta s}{\Delta t} = \frac{s - s_i}{t - t_i} \Rightarrow s(t) = v(t - t_i) + s_i$$
  $$\Rightarrow s(t) = v\Delta t + s_i$$

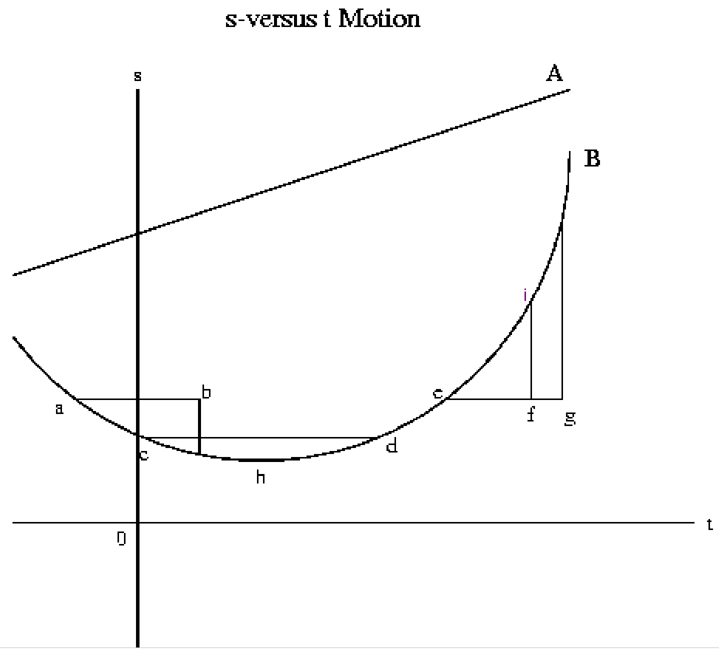
- Rearranging a definition produces an equation
- Here, position is seen to be a function of time
- Here, i refers to “initial”, f refers to “final”, but more general future event without subscript
- Here, v is a constant (the uniform rate of motion) and s_i is also a number
- By designating the parameters in this way, one can use the point-slope approach to find an equation of the motion—and then extrapolating to arbitrary time (including where the object was at t = 0 and even clock readings before the initial time)

Position vs Time Graph for Uniform Motion



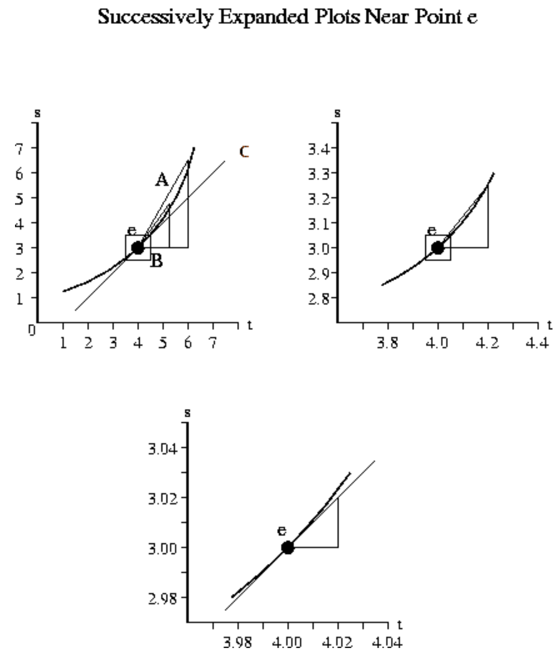
- Notice, uniform velocity is a straight line on a position versus time graph
- The slope of a position versus time graph of uniform motion is the uniform (constant) velocity
- Sign of the slope indicates direction, while the magnitude (quantifying the steepness of the slope) indicates the rate
- Positive velocity is always greater than negative velocity, even if the magnitude of the negative velocity is greater than that of the positive velocity; similarly, less negative is greater than more negative; zero > negative
- Notice that, here, $s_i = s_0$, so $t_i = 0$, and therefore Delta t = t (not the instant t, but the time interval from t = 0), and the geometric representation, then, of vt: the displacement since t = 0
- Problem 10: practice graphing straight lines and interpreting the parameters

Uniform vs Nonuniform Motion



- Line A is a position versus time graph of uniform (constant) motion: the slope is the same over any time interval
- Curve B is a position versus time graph of non-uniform motion: the slope differs everywhere, eg., less negative approaching point h from the left and increasingly positive to the right (the average velocity between points c and d is zero)—the slope is systematically increasing; the object’s velocity is always increasing
- The average changes everywhere, no single number can characterize the rate of motion, which requires a different number at each point—instantaneous velocity (velocity at an instant)

Slope of a Curve at a Point



- By finding the average velocity ever closer to a point on the curved line, the result is ever closer to the tangent line, which is the curve's slope at a point
- Problem 11: practice taking limits on a curve

Refining and Extending "Velocity"

- Instantaneous velocity: $v(t) \equiv \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} \equiv \frac{ds}{dt}$
  - Slope of the line tangent to the *s*-*t* curve (and thus the slope of the curve) at some instant
  - The uniform motion that would ensue if velocity change ceased at the instant the instantaneous velocity is being determined
- Average velocity: the uniform motion that results in the same displacement in the same time interval as the actual motion

- In the limit, as the time interval approaches zero, the slope is that of the tangent (of the curve) at that clock reading (instant)
- The derivative is the slope of the tangent at a single point of a curve (assuming it is defined there) and thus the slope of the curve at that point
- Problem 12: Practice visualizing the slopes of s-t curves and interpreting them as instantaneous velocities

Velocity vs Time Graphs

- Instantaneous velocity = velocity at some instant
- (t_j, v_j) can be plotted to give a "history" of motion
- Change of velocity: $\Delta v \equiv v_2 - v_1$
  - Sign of Δv indicates direction (not magnitude) of change

- Again, ordered pairs of numbers (here, clock readings and instantaneous velocities), graphed on a scatter plot can offer a visual record of the motion
- Problem 13: practice interpreting v-t plots
- v vs t graphs are very informative