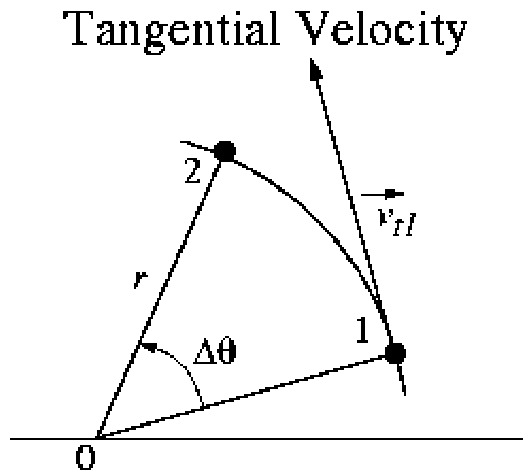
Motion: Week 4

Sections 4.31 – 4.38, pages 49 - 63

- Relationships between linear and angular kinematics
- Relative motion
  - Relative velocity
  - Reference frames
  - Transformations between reference frames

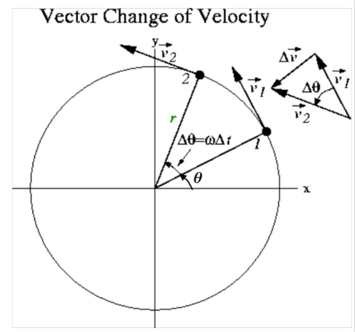
Relationship Between Linear and Angular Velocities

- $v_t = r\omega$
  - For a given angular velocity, the tangential velocity depends on distance from the center of rotation
- $a_t = r\alpha$

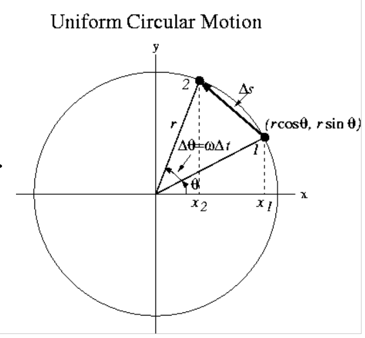


- In circular motion, instantaneous linear velocity (slope of the trajectory) is tangential and proportional to angular velocity and distance from the center of rotation
- For a velocity to change magnitude only (not direction), the acceleration must be (anti-)parallel to the motion
  - In the case of non-uniform circular motion, this means the acceleration is also tangential
  - Since r is constant, if the tangential velocity changes magnitude, then angular velocity must change its magnitude: angular acceleration
- In circular motion, the instantaneous tangential acceleration is proportional to the angular acceleration and the distance from the center of rotation
- For uniform circular motion, in which r and omega are constant, alpha and therefore the tangential acceleration are zero, and, hence, the magnitude of the linear (tangential) velocity is constant
- Problem 37 and 38: Analyze the motion of a wheel
- Problem 39 and 40: Find tangential velocities (latitudes, satellites)

Linear Acceleration in Uniform Circular Motion

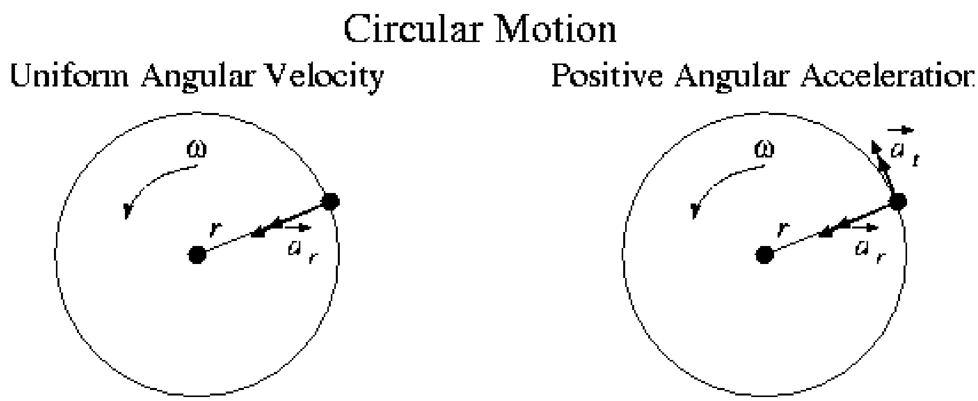


- $x = r\cos\theta,\ y = r\sin\theta$
- $\vec{v} = (v_x,\ v_y) = \left(\frac{dx}{dt},\ \frac{dy}{dt}\right)$
- $v_t = \vec{v} = (-y\omega,\ x\omega)$
  - Perpendicular to $\vec{r} = (x,\ y)$
- $a_r = \vec{a} = (-x\omega^2,\ -y\omega^2)$
  - Perpendicular to $\vec{v}$, but antiparallel to $\vec{r}$
- $v_t = r\omega \Rightarrow a_r = r\omega^2 = \frac{v_t^2}{r} = v_t\omega$



- While its magnitude may not change [there's no acceleration (anti-)parallel to it, the tangential (instantaneous) velocity's direction changes continuously
  - Any velocity change, whether magnitude or direction (or both) implies an acceleration
  - Since only direction—not magnitude—changes here, the acceleration must be perpendicular to its direction
- Direction can be found with components
- Components can be found with analytic geometry and trigonometry or with the Calculus using the chain rule
  - The result for linear velocity is, in fact, perpendicular to the radius—i.e., tangential to the motion
  - The acceleration is antiparallel to the radius, called radial/centripetal acceleration, and perpendicular to the tangential velocity, so changes only direction, not magnitude
- Problem 41: Make substitutions to justify equation 70

Acceleration in Circular Motion



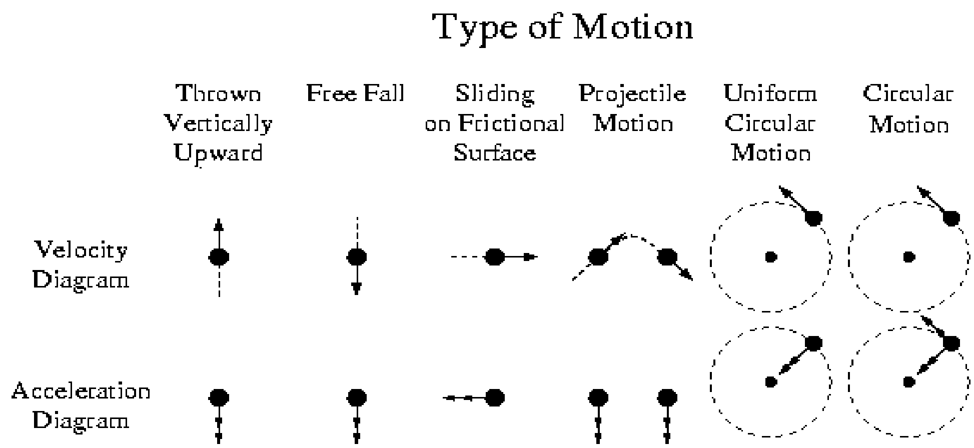
$$a_r = r\omega^2 \qquad a_r = r\omega^2$$

$$a_t = 0 \qquad a_t = r\alpha$$

$$\vec{a} = (a_r, a_t)$$

- Double-headed arrow to distinguish acceleration from velocity

Velocities and Accelerations of Different Motions



- Even if motion is changing, rarely are acceleration and velocity in the same direction
- In projectile motion, a uniform acceleration exists along one Cartesian axis and the magnitude and direction of the linear velocity changes continually
- Circular motion requires centripetal acceleration, anti-parallel to the radius and perpendicular to the tangential velocity.
- If the radial distance undergoes change, a non-zero radial (velocity/acceleration) component exists and the motion would no longer be simply circular
- If the radial velocity changes with time there’s an additional radial acceleration (in addition to the centripetal acceleration r omega^2) dv_r/dt
- Problem 42 and 43: Calculate centripetal accelerations for various curved motions (note circles at different latitudes)

Relative Motion

- Position and velocity are measured with respect to a reference frame
  - Reference frames may be observed (and measured) to be moving
  - All positions and velocities are “relative”
- Another application for which vectors are useful

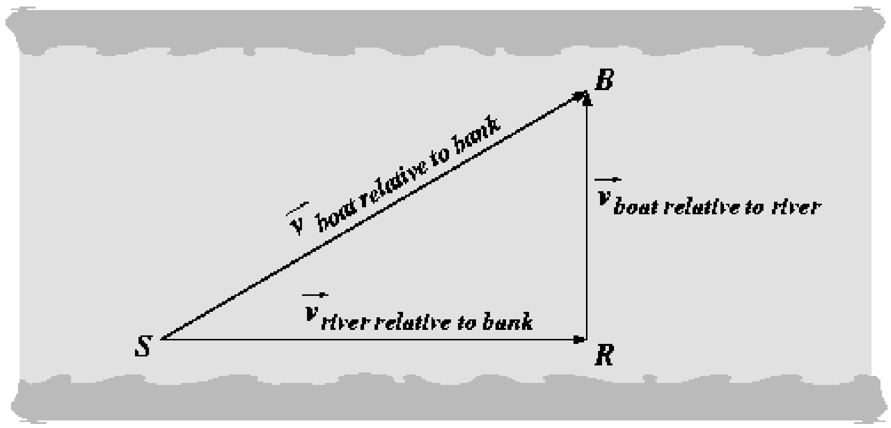
- Problem 44: Thrown stone viewed from a helicopter rising at constant velocity (the same velocity as the stone at release); qualitative analysis of relative motion

Relative Velocity

- Relative velocity: the velocity measured with respect to a reference frame observed to be moving
- Vectors—carefully done—can help understand relative motion

- Problem 45: kinematic relations in a moving reference frame; or of a moving reference frame
- Problem 46: kinematic relations of two objects in uniform motion

Relative Velocity



- Describe what's going on
- Problem 47: variation on river crossing, but rower wants to go straight across; read problem carefully and choose a convenient axis

Reference Frames

- An inertial reference frame is one that is not undergoing any kind of acceleration (moves uniformly—including remaining at rest)
- No experiment can differentiate one inertial reference frame from another inertial reference frame
  - The laws of physics hold identically in any laboratory moving at constant rectilinear velocity
  - Relativity principle: certain relationships are invariant when measured in inertial frames

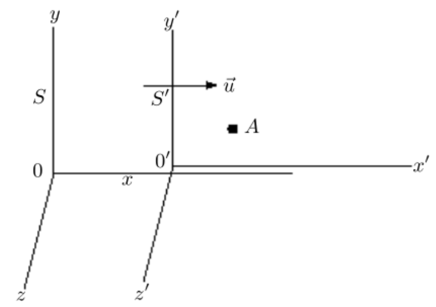
- A velocity measurement depends on the relationship of the measurer to the object whose velocity is being measured and, thus, the reference frame
- In an inertial frame, an object at rest remains at rest, and an object moving in uniform rectilinear motion continues to move in a straight line at constant velocity, unless acted upon by something external to the object —law of inertia or Newton's 1st law

Transforming Between Reference Frames

- Measure in the most convenient frame
- Galilean/Newtonian velocity transformations:

$$v_x = v'_{x'} + u$$
$$v_y = v'_{y'}$$
$$v_z = v'_{z'}$$

Two Inertial Reference Frames



- Transformation: relationship between numerical values describing motion or other physical phenomenon measured in different reference frames

Length and Time in a Single Reference Frame

- Length measured with a scaled rigid rod
  - Positions ⇒ displacements, velocities, accelerations
- Instants (clock readings) measured by counting repetitions of periodic processes
- Event: simultaneous coincidence of a position number with a clock reading
- What is meant when two widely separated events are said to take place simultaneously?

- Nothing to add

Synchronization

- To accept that two events separated in space were simultaneous, clocks at the two locations must be assumed to be synchronized
- To synchronize separated clocks, a signal can be sent
  - Velocity of signal (c) and distance signal travels ($\sim L$) must be known
  - $\bar{v} = \frac{2L}{\Delta t} \equiv c$
  - Setting a time scale requires defining a position scale

- One idea: set them to the same time at one position and then move them to another position—assumes clocks are unaffected by being moved
- Alternatively,
  - measure the time it takes a signal reflected from a mirror at a known distance away from a single clock to return, and thereby determine signal velocity
  - Assume all other clocks at all other positions are identical
  - Set clocks ahead the signal propagation time from a central source and start them when the signal arrives
- Space and time inextricably intertwined
- Problem 48: Synchronize clocks on Earth and moon