

HISTORY OF PHYSICS

by MAX VON LAUE

translated by RALPH OESPER



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The Reference System of Physics

THE PROBLEM, suggested in the chapter heading, can be traced back into Greek antiquity. It had two epochs: the geometrical, which lasted into the seventeenth century; and the dynamic, which, after the triumph (around 1800) of the wave theory of light, spread out until it embraced all of physics.

In the geometrical period, the question of the reference system was directly linked with the nature of the location and motion of a body. From the very beginning, it was apparent that both concepts were without meaning in the absence of something to which they could be referred. Thus Aristotle, and with him all of the scholastics, located a body on a material structure which embraced it; moot points being whether this structure had to be in immediate contact with the body or whether finite distances between them were allowable. Disputes arose over such questions as to whether a ship anchored in a stream moves when the wind blows, since the water and air in the ship's immediate vicinity constantly change, or whether the vessel is at rest, since it exhibits no movement that can be seen from the bank. Such questions could not be decided under the prevailing conditions. From the standpoint of physics, it was more important when Claudius Ptolemy, who lived at Alexandria in the second century A.D., declared that the sphere of the fixed stars, the outermost of a series of spheres surrounding the earth, had no location whatsoever, there was nothing at all beyond it, not even space, which should have enveloped it. It is a quite remarkable feature of the Ptolemaic system that it insisted that this sphere nevertheless possessed motion, i.e., a daily rotation around the earth.

Nikolaus Copernicus noted this inconsistency when he

founded the astronomical system which bears his name. When he declared that the sphere of the fixed stars remains at rest, while on the contrary the earth rotates daily about its axis, this was, at first, no more than an improvement of the consequence of the idea that had come down to him. He also firmly retained the notion of the existence of utter void beyond this sphere, an idea which the moderns find exceedingly difficult to comprehend. The first to free himself of this notion was the powerful attacker of Aristotle, Giordano Bruno, who in 1600 was burned at the stake at Rome as a penalty for teaching the related doctrine of the unlimited multiplicity of worlds, his defense of Copernicus, and similar heresies. Not even a man of Kepler's stature was able to adopt this bold and yet so unavoidable addition to the Copernican system. In the further establishment of the heliocentric system, Copernicus allowed himself to be guided by the, so to speak, teleological viewpoint of the simplicity of Nature; even the Greeks were familiar with the phrase "Nature does nothing superfluous or in vain." He found it simpler to explain the occasional retrograde motion of the planets in the heavenly vault as due to the movement of the earth itself, i.e., of *one* body around the sun, rather than through the motions compounded of several circular movements of *all* the planets, as postulated by Ptolemy. Though this thought is congenial to the modern scientist, the state of physics at that time was such that nobody could provide a causal basis for it. Though appealing more to sentiment than to reason, this demand for simplicity was incapable—as is entirely understandable—of convincing many of his contemporaries and those who came later, principally because the idea that all humanity was being whirled around in a circle without being conscious of the fact was not exactly simple and, in addition, at that time without foundation in physics. Hence, by no means need it to have been mere backwardness or anxiety which impelled Osiander, the Nuremberg scholar, who supervised on the very spot the publication of the great Copernican treatise "De revolutionibus"¹ to state in the Foreword, which

¹ The last two words in the title "De revolutionibus orbium coelestium" were added by the publisher himself.

he supplied to the work, that the Copernican system was being presented as a pure "hypothesis," which, although it was in accord with the observations and was therefore justified to that extent, was by no means necessarily "true" because of this agreement. The heat of the controversies which this system quickly engendered likewise was due, in no small measure, to the total lack of physical-causal reasons, either for or against it, during the entire "geometrical" epoch. An additional factor, of course, was the opposition of the representatives of the church, Protestant as well as Catholic. They, Dr. Martin Luther, for instance, denied the movement of the earth as being contradictory to the Bible. Actually they were only repeating the indictments, that had been leveled in the third century B.C. against the first champion of such a system; Aristarchus of Samos had likewise been labeled "scoffer at religion." Experience has revealed a very singular fact, namely, that—from Aristarchus to Einstein—nothing physical embitters wide circles of the public so much as an attack on firmly entrenched concepts of space and time. Similarly, Galilei's condemnation was not connected with the geometrical arguments which Copernicus had to cite for his system nor with Galilei's astronomical discoveries, which he utilized in its support, but rather he was condemned because of the "Dialogue on the Two Chief Systems of the World," which refutes the dynamic reasons against the movement of the earth in detail and sometimes caustically.

Copernicus himself was not touched by the disputes concerning his system. He had held back his book, begun in 1507, until 1543, and at most saw parts of it in print on his death-bed. To be sure, much earlier, probably in 1514, he had sent a friend a sort of preliminary announcement in manuscript form. This later became known as the *Commentariolus*; after being forgotten for many years, it turned up in the Vienna Library about 1880. The following rules are taken from it:

1. There is not merely one central point for all heavenly circles or spheres.

2. The center of the earth is not the center of the world, but only of that of heaviness and the orbit of the moon.

3. All orbits surround the sun, as though it lies in the middle of all, and consequently the center of the world lies near the sun.

Obviously to meet the objection that the appearance of the heaven of the fixed stars must change in the course of a year because of the terrestrial motion, there then follows:

4. The relation of the distance sun-earth to the height of the fixed star heaven is smaller than that of the earth's radius to its distance from the sun, so that this is imperceptible in comparison with the height of the fixed star heaven.

5. All that is visible of motion in the fixed star heaven is really not within itself, but is seen from the earth. Hence, the earth, together with its contiguous elements, rotates once each day about its invariable poles. Meanwhile, the fixed star heaven remains immovable as the outermost heaven.

6. All that is visible of motion on the sun does not arise through itself, but because of the earth and our circular orbit, with which we, like every other planet, revolve around the sun. And thus the earth is carried along by several motions.

7. The seeming retreat and advance of the planets is not within them, but are seen from the earth; its motion therefore is sufficient to account for so many varied phenomena in the heavens.

This now appears to be clear and simple, and yet the conception of such a system still gave Copernicus trouble. How otherwise can it be explained that, in addition to the daily and yearly rotation of the earth, he later also ascribed to it a third motion? This was supposed to account for the fact that the earth's axis changes its position relative to the sun during the course of the year, i.e., it was to explain the change of the seasons. It would have sufficed, however, if the earth maintained its position relative to the heaven of the fixed stars. This error is somewhat reminiscent of the controversy current in Newton's time as to whether the moon has a rotation of its own,

since it always presents the same side to the earth. Those who denied this rotation simply could not free themselves completely from their ingrained geocentric viewpoint. A certain prejudice in favor of the sun certainly played a similar part in the case of Copernicus.

Be that as it may, we owe to Copernicus the reference system, which rests in the center of gravity of our solar system, with its axial directions oriented toward the heaven of the fixed stars. Physics refers all locations and all motions to this system, if nothing is stated to the contrary. The three coordinates, by which the point is defined in terms of Cartesian analytical geometry, in physics are referred to this system, if they are not expressly defined in some other manner. Neither Kepler's laws nor the gravitational theory would have been discovered without Copernicus. It is conceded that its foundation remained incomplete; newborn sciences cannot be established in their entirety from the very start. The greatness of their creators is revealed precisely in the fact that they, nonetheless, intuitively hit upon the right thing.

Johannes Kepler contributed to the consolidation of the Copernican system to the extent that his three laws of planetary motion, and the more exact computation of observations which they made possible, had been nurtured wholly in its soil; furthermore, they could scarcely have fitted into the Ptolemaic system. However, Kepler had little inclination to establish the Copernican system more firmly; his arguments in its favor go not much beyond pointing out its simplicity and beauty, features which Copernicus himself had thrown into the scales. Although Kepler received the reports of the astronomical discoveries of Galilei with delight, he seemingly had little understanding of his contemporary's contributions to dynamics, despite their importance in deciding between Copernicus and Ptolemy, and in the face of the fact that dynamical arguments against the Copernican system had quickly been raised by its opponents. They stated, for instance, that the daily rotation of the earth around the sun would surely cause everything that was not securely fastened to the earth to be hurled into space,

and that such articles because of the motion around the sun would be brushed off by the earth and so trail behind. Objections of this type could only be successfully met by the new dynamics. The decision came partly from Galilei, and the matter was finally settled by Newton.

In his time, Galilei was the most successful, most popular, and most hated champion of the Copernican system. His contribution lay first of all in his astronomical discoveries which were made possible by the use of the new tool, the telescope. In 1610 he showed that Jupiter and its satellites are a Copernican system in miniature. In 1611 he definitely proved by means of the phases of Venus that this planet follows an (approximately) circular orbit around the sun and that it, like the earth and the moon, does not emit any light of its own, but merely reflects the light falling on it from the sun. The ashen-gray light sent out by the part of the moon not illuminated by the sun was proof to him that the earth, if viewed from without, would appear bright like the other planets. Secondly, he had the clear insight to comprehend that the laws of motion should not be related to an earth-bound reference system, but to the Copernican system. Although when discussing the falling of bodies, he usually spoke of motion toward the earth, he nevertheless added that this is only approximately permissible, that in the strictest sense, a freely falling body deviates from the vertical because of the rotation of the earth. This approximation, whose limitation is fully recognized, is still employed by physicists in the discussions of most of their experiments. Galilei never tired of using his newly acquired knowledge in refuting the dynamic objections against Copernicus. However, Newton's work made it perfectly evident that the planetary motions can be understood dynamically—but then also completely—only when the Copernican reference system is taken as the basis.

The problem of the reference system had thus been decided practically, but not fundamentally. By what physical authority is the Copernican system of reference given precedence over all others, an earth-bound one, for instance? Newton, who was

thoroughly aware of the gravity of this question, had recourse to the assumption that there is an "absolute" space, just as there is an "absolute" time, and that it precisely defined the correct reference system. Most people, however, are inclined to agree with Ludwig Lange (1863-1936), who found both concepts hard to grasp and rather "ghostly"; nonetheless, they still haunt some minds today. As a matter of fact, the debatableness of this idea gave rise to thought by all the great philosophers from Newton's time on—Leibniz and Kant, for instance. But the liberating word did not come until 1886 when Lange's "The historical development of the concept of motion" appeared. He wrote: "Physics defines its reference system according to the purpose which it is to fulfill, hence, from the same viewpoint which is also the basis of measuring time." Lange summarized the result of his deliberations in two definitions and two theorems.²

Definition I: An inertial system means every coordinate system of such nature that with reference to it the paths of three mass points (which, however, may not lie on a straight line) projected from the same point in space and then left to themselves are all straight lines.

Theorem I: With reference to an inertial system the path of every fourth point left to itself is also a straight line.

Definition II: Inertial time scale means every time scale with respect to which a mass point left to itself traverses equal distances in equal times in its inertial path.

Theorem II: With respect to an inertial scale, every other mass point also traverses equal distances of its inertial path in equal times.

The definitions are human convention, but the theorems are empirical propositions, and only through them do the definitions acquire value for physics. The truth of the Copernican system lies in the empirical validity of the theorems.

Of course, it was not possible to conclude from the observation of force-free mass points that the Copernican system is an inertial system. However, a perfectly valid proof of this

² The quotations are from a section of his *Philosophischen Studien*.

resides in the agreement with experiment that is obtained when planetary orbits are computed on the basis of mechanics which contains the law of inertia. It was pointed out in Chapter I that, in order to attain nicety in this agreement, certain slight corrections must be applied to the ordinary time scale, so as to transform the latter into the inertial scale.

The work of Ludwig Lange thus finally brought a certain degree of conclusion to the development, which was begun three and a half centuries previously by Copernicus. The fact that the other natural laws, optical, electrical, etc., can be simply formulated in the same system, is, of course, a further purely empirical fact.

Many other conceivable reference systems are excluded by the above requirement of serving the purpose; for example, every system that rotates with constant velocity counter to the astronomical system. A body at rest in such a system—even Newton called attention to this possible case—would apparently experience a centrifugal force, of which the equations of motion contain nothing, and which actually is nothing more than another expression for the tendency toward linear motion in relation to an inertial system. In the case of the earth-bound reference system, which therefore rotates with the earth, this is manifested, for instance, by the flattening at its poles. The rotation of the plane of the swing of the so-called Foucault pendulum, which was first demonstrated in public in 1851, gives further incontrovertible evidence of the earth's rotation. In other words, this again proves that the fixed-on-earth reference system has no justification, as does the fact that the gyroscopic compass takes a north-south position. A. A. Michelson furnished another proof in 1925 by means of an optical interference experiment.

Still other systems, however, can be derived from one inertial system according to dynamics. All reference systems have a standing equal to that of any other system toward which they possess a constant translation velocity. Newton was well aware of this. In fact, Galilei, when defending the Copernican theory against popular mechanical objections, emphasized that no

mechanical experiment will reveal the motion of a moving ship to a passenger inclosed within the vessel. The coordinates of a mass point in one system can be calculated by means of a simple formula from the coordinates with reference to the other. Time also enters in this process, of course, corresponding to the relative motion of both systems. It remains untransformed, and in this sense is "absolute." The velocities of the mass points are different in two such systems, but not their accelerations; consequently, the law of motion holds for both in exactly the same form. If there were a single inertial system, it could be put down as the absolute reference system, and motion with respect to it could be termed "absolute" motion. Since this is not so, one speaks of the relativity principle of the Newtonian mechanics. The meaning of this principle is that mechanical experiments cannot reveal the superiority of one inertial system over another inertial system.

For a long time it was necessary to conjecture that perhaps other experiments and observations might permit this, namely, all those in which physical effect are propagated with finite velocities, since the velocities are different in different inertial systems. This idea became especially important for optics. Every theory which required an ether as carrier of the light—and eventually the electronic theory came to it (Chapters IV and V)—must, of course, regard the reference system in which it is at rest as superior to all others. It thereby defines an absolute reference system. That this is an inertial system in the sense of mechanics was always tacitly assumed.

Actually, the aberration of the stars, discovered in 1728 by J. Bradley (Chapter IV), could be simply explained on the basis that in the astronomical reference system the light travels in all directions with the same velocity which had been determined by Olaf Römer, and that its relative velocity with respect to the earth can be obtained from this through vectorial subtraction of the velocity of the earth. The fact, which seemed paradoxical at first, that filling the telescope with water, i.e., changing the speed of light within the tube, would have no effect on the aberration was predicted in 1818 by Fresnel and

verified by an actual trial in 1871 by George Biddell Airy (1801-1892). Fresnel's theory is no longer of interest, but it led to the correct result, verified in 1851 by A. Fizeau, for the velocity of propagation of light in moving bodies. If the ray and the motion of the body have the same or opposed directions, the entire velocity of the body is not added to or subtracted from the velocity of the light with respect to the body, rather it must be supplied with a reducing factor, the Fresnel entwinement coefficient. This served as a valuable touchstone for all later theories of the optics of moving bodies. The Fizeau experiment was long held to be a decisive proof of the existence of an ether, which was supposed to permeate all bodies without participating in their motion. Only in this way could this reducing factor be understood. It was reserved for the relativity theory to disprove this argument. It pointed out that the addition or subtraction of velocities, hitherto taken as self-evident, is not justified under the conditions prevailing here. Consequently, the history of the Fizeau experiment is an instructive illustration of the extent to which even theoretical elements enter into the explanation of every experiment; they cannot be excluded. Accordingly, if theories change, what has been an impressive proof of the truth of one of them can easily become an equally strong argument in favor of one that is quite different.

The old notion of the additivity of the velocities of light and moving bodies, however, found verification in other connections. For example, in 1842 Christian Doppler (1803-1853) deduced from the undulatory theory that the observed vibration frequency increases as the light source and the observer approach each other, and decreases as they get farther apart. Although it is difficult to understand why, it is a fact that this Doppler principle, despite its acoustical confirmation (Chapter II), was bitterly opposed for several decades. The reason, in part, stemmed from Doppler's untenable applications of it to astronomy. Yet he was correct in so far as astronomy was the first field in which substantiation of the principle was obtained. In 1860 Ernst Mach (1838-1916) stated clearly that

those absorption lines in the stellar spectra that come from the stars themselves must show the Doppler effect; he likewise pointed out that these spectra contain other absorption lines of terrestrial origin which do not show this effect. The first pertinent observation seems to have been that by William Huggins (1824-1910) in 1868. Today the accuracy, under favorable conditions, of such observations is so high that radial velocities of 3×10^4 cm/sec can be detected, whereas some up to 10^7 cm/sec occur. The laboratory proof of the Doppler effect was obtained in 1905 by Johannes Stark. He worked with canal rays, i.e., luminous atoms, which in electrical gas discharge acquire velocities up to 10^8 cm/sec, so that the Doppler shifts of the spectral lines are incomparably greater than in astronomy. In 1919 Q. Majorans verified the effect by means of mechanically moved light sources with velocities of about 2×10^4 cm/sec.

No matter how great the importance of aberration and Doppler effect, they do not answer the question as to the existence of several optically justified reference systems. More intimate studies show that they are not concerned with the velocities of the light source and the observer with respect to a reference system, but only—at least in first approximation—with the relative velocity of the two with respect to each other. On the other hand, would the existence of a preferred reference system be proved, if an observation, with all the participating bodies moving at the same velocity, should reveal an effect on this velocity? Under such circumstances, this velocity would be in competition with the velocity of light; the ratio of the two velocities is involved, and this is always a small number. Consequently, such observations are difficult, even when they involve an effect of not more than the first order, i.e., proportional to this ratio itself, and are exceedingly so when an effect of the second order enters, i.e., when the square of the ratio is involved. It is hopeless to attain adequate velocities of all the participating bodies in the laboratory. The velocity with which the earth revolves around the sun is necessary for this purpose; but even in this case the ratio equals

10⁻⁴. The object of such experiments is to prove this orbital velocity or, in other words, to demonstrate the "ether wind" with respect to the moving earth.

Attempts to do this have been made ever since 1839, when Jacques Babinet (1794-1872) sought to discover an influence of the earth's motion on interference phenomena. The results invariably were negative. Most of the experiments involved effects of the first order and could no longer be applied in deciding the question regarding the reference system after H. A. Lorentz in 1895 proved, by means of the electron theory, that there can be no such optical or even electromagnetic effects of the first order. Hence, the few experiments which involve effects of the second order became all the more significant. Among these none is easier to understand as to its underlying theory and more certain in its experimental accomplishment than the Michelson experiment. It gives a direct comparison of the relative velocities with which light travels in different directions with respect to the earth. The ether wind, if it exists, would have produced differences between these velocities.

The idea, and the first, though inadequate trial was published by A. A. Michelson in 1881. After Lorentz had pointed out its defects in 1884, Michelson and E. W. Morley repeated the experiment in 1887 with modifications that provided the required accuracy. In 1904 Morley and D. C. Miller advanced much farther; they reported that the observed effect did not amount to even one one-hundredth of the expected result. Although after 1920 Miller thought he had obtained positive results at great altitudes, they were contradictory among themselves and furthermore were disproved by repetitions carried out in 1926 by I. R. Kennedy and by several measurements made in 1926-27 by A. Picard and E. Stahel, some on Mt. Rigi and some in a balloon. The accuracy was enhanced so much by K. K. Illingworth in 1927 and by G. Joos in 1930 that an "ether wind" of 1 or 1½ km/sec must have made itself evident, if the theory of the preferred reference system were correct.

The influence of the Michelson and several other similar experiments gave rise to the special relativity theory. This ushered in a new era for the problem of the reference system. It asserts as a natural law the existence of an infinite number of inertial systems, which have translatory motion with respect to each other of constant velocities, and are equally justified for the *totality of natural events*. To be sure, when converting from one to the other, it is not possible to proceed as in Newtonian mechanics, namely, the time is not transformed at all and all material distances are left as they were; therefore the mechanics also requires the modification mentioned in Chapter II. To express the fact that light is propagated with the same velocity in all directions in every inertial system, as indicated by the Michelson experiment, it is necessary rather to have a simultaneous transformation of the local coordinates and the time. This "Lorentz transformation" leads, for example, to the law that every body which is moving against an inertial system is shorter in this direction than when it is at rest. Admittedly, this contraction is slight, of the second order for low velocities; but when the velocity approaches that of light, the shortening becomes very important. Indeed, the measuring of the body in this direction must sink below every limit, when the speed of light is closely approached. Another consequence is that the velocity of light becomes the upper limit not only for all corporeal velocities and for the propagation of all physical effects through space, but also for the relative velocities of all inertial systems with respect to each other. Accordingly, the velocity of light goes beyond the bounds of optics and electrodynamics and attains universal significance for natural events. It was a sort of historic accident that humanity first discovered this in the case of light.

The fact that this transformation from one optically justified reference system leads to another which is equally justified was present, in essence, in a study made by Waldemar Voigt (1850-1919) in 1887. It was explained around 1900 through in-

genious tentative ideas by Henri Poincaré. In 1904 it was confirmed, with the aid of electrodynamics, by H. A. Lorentz, who even then contributed the relativistically modified mechanics (Chapter II). Nevertheless, the viewpoint of all these forerunners remained that electromagnetic and optical processes occur *as though* this transformation leads again to a justified reference system. Lorentz, for example, differentiated in definite terms between the actual absolute time, which can be used directly for a justified reference system, and the "local times," which can be calculated from it and the local coordinates for other reference systems. The decisive change, the omission of the "as though," was made in 1905 by Albert Einstein. On the strength of a deeper insight into the essence of the measurement of space and time, he announced the *complete* equal validity of all reference systems derived by this transformation from a valid system, and therefore also the equivalence of all space and time measurements appertaining to them. The polemic against the relativity theory, in part, went beyond all reasonable bounds; it arose because many of the opponents lacked the requisite keenness of insight. This fundamental inversion then led its originator to the crowning glory of the whole structure of the relativity theory, namely, the law of the inertia of energy (Chapter II).

A material luminiferous ether is incompatible with the relativity theory; as was stated, it results in the preference for a particular reference system. With this, the Faraday-Maxwell concept of the electromagnetic field as a modified condition of the ether is eliminated; nothing remains except to regard this field itself as a reality.

Another conclusion drawn from the Lorentz transformation is that a clock in motion runs slower than when at rest. However, those periodic vibrations in atoms which produce the light of the spectral lines can be regarded as a "clock." Of course, this effect is small, of the second order, and accordingly difficult to perceive. However, the velocities of the canal rays sufficed to reveal it. In observations of the Doppler effect, it

superimposes itself on the familiar Doppler shift as a "quadratic Doppler effect," and this was actually observed in 1938 by H. I. Ives and G. R. Stievel, and by H. Otting in 1939.

The special relativity theory, which was discussed here, formed the close of a development which had been proceeding through a century. Precisely for this reason, it did not present experimental investigation with any new problems. Whatever new information has since been given by appropriate studies has come from improvements of earlier experiments. It has been pointed out to what extent these repeated the Michelson experiment, but it is useful to call attention also to an electrical-mechanical experiment devised and carried out in 1903 by T. Trouton and H. R. Noble, and refined in 1926 by R. Tomaschek. The total accuracy of the Michelson experiment was thus increased. According to the electronic theory, when a charge is given to a rotatable, suspended electrostatic condenser, it should turn because of the motion of the earth, whereas, according to the relativity theory, no rotation would ensue. The calculated effect would admittedly be slight, of the second order, but these workers were able to prove that it is not present.

The mathematicians and theoretical physicists found themselves with all the more to do. They had to adjust all branches of physics to the relativity theory, i.e., such fields as hydrodynamics, the elasticity theory, thermodynamics, and the parts of the Maxwellian theory that relate to matter. The relativity theory owes its elegant mathematical form to Hermann Minkowski (1864-1909) who, shortly before his death, introduced time as the fourth coordinate, with a validity equal to that of the other three. However, this addition involves nothing more than a very valuable artifice; it does not connote anything deeper, even though some tried to read a more profound meaning into it. Max von Laue brought out the first comprehensive presentation of the special relativity theory in 1911.

Einstein did not stop with the special relativity theory. The conditions with respect to the measurement of space are like those surrounding the measurement of time (Chapter I). A

continuum is presented to our intellect, and it is necessary to introduce a system of measures; but this problem is more involved because of the three dimensions of space. In principle there are an infinite number of equally valid methods for this. The mathematicians make the most of this multiplicity when they freely invent non-Euclidean geometries. Physics, however, is obliged to restrict this multifariousness because of the practical requirement that its geometry must make possible a *simple* presentation of the natural laws. This is the core of the question as to which geometry holds *empirically*. When, for instance, Gauss, in order to test the validity of Euclidean geometry, determined by geodetic, i.e., optical, methods, whether the sum of the angles of the triangle formed by three peaks (Brocken, Inselberg, Hoher Hagen) actually amounts to 180° as required by this geometry in contrast to the others, he tacitly required in interest of this simplicity that the rays of light follow geodetic ("shortest") lines. Whoever disclaims this can draw no geometrical conclusions at all from the experimental result confirming this sum of the angles.

Whereas physics, up to that time, had been able to get along perfectly well with Euclidean geometry, Einstein's general relativity theory, as it had gradually developed from 1913 on, believed that it would be forced to draw upon a non-Euclidean "Riemann" geometry. The deviations from the Euclidean are minimal, even in the vicinity of masses as large as the sun, and appear in only very few observations. These are (1) the advance in the perihelion of Mercury, which could not be explained by the planetary theory (Chapter III); (2) the change in direction of light close to the sun, which Arthur Stanley Eddington (1882-1944) found during the 1919 solar eclipse to correspond quite closely to that predicted by Einstein, while later eclipse observations, of course, produced a somewhat greater value. The third instance of the verification of the theory is drawn from the recent spectral studies of a particularly dense star, the companion of Sirius. The spectral lines from it are shifted considerably toward the red as compared with their position in a terrestrial spectrum. The case concerning this theory is not

yet closed; but it will always bear the honor of having predicted the deflection of light without special *ad hoc* assumptions.³

³ Others had predicted such deflections of light, but with such assumptions.