ELEMENTARY PARTICLES AND SU(4) *

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Recently, models of strong interaction symmetry have been proposed [1-3] involving four fundamental Fermion fields ψ_i and approximate symmetry under SU(4). Mesons are identified with bound states $\bar{\psi}^i \psi_j$ and baryons with bound states $\bar{\psi}^i \psi_j \psi_k$. In this note we examine a model of this kind whose principal achievements are these: a mass formula relating the masses of the nine vector mesons and predicting a ninth pseudoscalar meson at 950 MeV, a description of weak interactions including all selection rules except the nonleptonic $\Delta I = \frac{1}{2}$ rule, and a significant "baryon"-lepton symmetry. A new quantum number "charm" is violated only by the weak interactions, and the model predicts the existence of many "charmed" particles whose discovery is the crucial test of the idea.

We call the four fundamental "baryons" ψ_i = (Z^+, X^+, X^0, Y^0) and assume the strong interactions are *approximately* invariant under 4 × 4 unitary transformations. For convenience, we let this representation of SU(4) be the $\bar{4}$. We furthermore assume that the strong interactions are *exactly* invariant † under independent phase transformations of each of the four ψ_i and invariant under the isotopic group. (Z^+ and Y^0 are isosinglets and (X^+, X^0) an isodoublet). The four conserved quantum numbers we define to be baryon number B, charm C, charge Q and hypercharge Y, and their assignments are shown in table 1.

The eightfold way - possibly a more exact symmetry than SU(4) - is a subgroup of SU(4) corresponding to unitary transformations of the three fundamental charmed fields (Z^+, X^+, X^0). They transform under the SU(3) representation $\bar{3}$, while Y^0 is an SU(3) singlet.

Table 1
Quantum numbers of the fundamental fields.

	B	C	Q	Y	I	I_3
Z^+	1	1	1	1	0	0
X^+	1	1	1	0	$\frac{1}{2}$	$\frac{1}{2}$
X^0	1	1	0	0	$\frac{1}{2}$	$-\frac{1}{2}$
Y^0	1	0	0	0	0	0

We assume that the pseudoscalar mesons transform under the (adjoint) representation 15 contained in $4 \times \bar{4}$:

$$M_i^j = \psi_i \bar{\psi}^j - \tfrac{1}{4}\,\delta_i^j \psi_k \bar{\psi}^k \,.$$

These 15 mesons form four SU(3) submultiplets: a $C = 0$ singlet, a $C = 0$ octet, a $\bar{3}$ with $C = 1$, and a 3 with $C = -1$. They are conveniently displayed as a 4 × 4 matrix:

$$M = \begin{bmatrix} \left(-\frac{2}{\sqrt{6}}\eta + \frac{1}{\sqrt{12}}\chi\right) & K^0 & K^+ & S_p^+ \\ \bar{K}^0 & \left(\frac{1}{\sqrt{6}}\eta + \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{12}}\chi\right) & \pi^+ & D_p^+ \\ K^- & \pi^- & \left(\frac{1}{\sqrt{6}}\eta - \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{12}}\chi\right) & D_p^0 \\ S_p^- & D_p^- & \bar{D}_p^0 & -\frac{3}{\sqrt{12}}\chi \end{bmatrix}$$

The ninth pseudoscalar meson without charm is called χ. The charmed particles comprise an isosinglet S_p^+ with $C = Y = 1$ and an isodoublet (D_p^+, D_p^0) with $C = 1$, $Y = 0$, and their antiparticles with $C = -1$.

In analogy with Gell-Mann and Okubo [4], we obtain a mass formula if we assume that symmetry-breaking effects transform like a member of the adjoint representation of SU(4). Thus mass splittings may transform like C or Y. For pseudoscalar mesons the mass formula contains only three terms, and all masses are determined in

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† In this model $Q = T_3 + \frac{1}{2}(Y + C)$. Thus, any strong violation of "charm" satisfies $\Delta Y = \Delta C$. This leads to weak interactions with $\Delta Y = 2$, $\Delta C = 0$, which are incompatible with the small $K_1 - K_2$ mass difference.

terms of those of π, K and η. We obtain * (using the squares of meson masses) $m(\chi)$ = 950 MeV, $m(D_p)$ = 760 MeV, and $m(S_p)$ = 900 MeV. The ninth pseudoscalar meson χ is tentatively identified with the recently discovered $\eta\pi\pi$ resonance [6]. The charmed mesons have not been observed. They would be produced in strong interactions only in association with oppositely charmed particles. They would decay weakly.

The vector mesons are also assigned to the adjoint representation 15. The corresponding 4×4 matrix is obtained from that of the pseudoscalar mesons by the replacements $K \rightarrow K^*$, $\pi \rightarrow \rho$, $\eta \rightarrow \omega_8$, $\chi \rightarrow \omega_1$, $S_p \rightarrow S_v$, and $D_p \rightarrow D_v$. Again two triplets of oppositely charmed particles are required in addition to the usual nine vector mesons. The mass formula is of the same form, but we apply it to the inverse squares of the vector meson masses. This point of view has been advocated by Coleman and Schnitzer [7], and follows naturally in a gauge-invariant tadpole theory **. Fitting the three parameters to the masses of K*, ω and φ, we obtain $m(\rho)$ = 740 MeV, $m(D_v)$ = 770 MeV and $m(S_v)$ = 940 MeV. The ρ mass is correctly predicted and the masses of the charmed vector mesons come out (coincidentally?) very close to the masses of the charmed pseudoscalar mesons.

The physical particles ω and φ are mixtures of ω_1 and ω_8. In this m^{-2} formalism we obtain $\varphi = \cos\lambda\ \omega_8 - \sin\lambda\ \omega_1$ with $\lambda = 27^o$. The form of vecton-vecton-meson coupling is unique, so that $g(\varphi\rho\pi)/g(\omega\rho\pi) \approx \frac{1}{2}$ is determined by the mixing angle. The suppression of the decay $\varphi \rightarrow \rho\pi$ can be understood within this model only if all vecton-vecton-meson couplings are small (or if there is a 16th vector meson).

The baryon octet must be associated with one of the irreducible representations of SU(4) contained in $4 \times 4 \times \bar{4}$. The simplest possibility is the 20-dimensional representation obtained by antisymmetrizing and making traceless the expression $B_{ij}^k = \psi_i\psi_j\bar{\psi}^k$. Its SU(3) decomposition yields four sub-multiplets: the familiar $C = 0$ baryon octet, a $\bar{3}$ and 6 with $C = 1$, and a 3 with $C = 2$. The mass formula contains five terms, three giving the usual Gell-Mann-Okubo formula for the SU(3) submultiplets and two describing the splittings among them. The two new parameters may be chosen in such a fashion that the baryons are the lightest submultiplet. The charmed submultiplets may be made arbitrarily heavier.

The decuplet of $J = \frac{3}{2}$ meson-baryon resonances are uniquely accomodated in one of the representations contained in 15×20. It is a different 20-dimensional representation given by the symmetrization of $R^{ijk} = \epsilon^{klmn} B^i_{lm} M^j_n$. Its SU(3) decomposition yields a $C = 0$ decuplet, a $C = 1$ sextuplet, a $C = 2$ triplet, and a $C = 3$ singlet. The mass formula predicts equal spacing in charm as well as equal spacing in hypercharge.

A fundamental similarity between the weak and electromagnetic interactions of the leptons and the ψ_i is established with the correspondence [1,3]

$$\begin{aligned} \mu^+ &\leftrightarrow Z^+ \\ e^+ &\leftrightarrow X^+ \\ \nu &\leftrightarrow X^o \cos\theta - Y^o \sin\theta = X' \\ \nu' &\leftrightarrow Y^o \cos\theta + X^o \sin\theta = Y . \end{aligned}$$

Evidently the electromagnetic interactions of l_i and ψ_i are identical. Weak interactions (of the form $G\, J^\mu J^+_\mu$) involve a lepton current $\bar{e}^+\gamma_\mu(1+\gamma_5)\nu + \bar{\mu}^+\gamma_\mu(1+\gamma_5)\nu'$ and a "baryon" current which is obtained from the lepton current by the above substitution together with the replacement $\gamma_\mu(1+\gamma_5) \rightarrow \gamma_\mu(1-\gamma_6)$. The baryon current is thus $\bar{\psi}\gamma_\mu(1-\gamma_5)W\psi$, with the 4×4 matrix W given by

$$W = \begin{pmatrix} 0 & 0 & \sin\theta & \cos\theta \\ 0 & 0 & \cos\theta & -\sin\theta \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

The relation $[W, W^+] = 2Q - 1$, independent of θ, suggests a possible intimate connection between weak and electromagnetic interactions †. Note also that each of the three basic interactions involves just four of the eight fields ψ_i and l_i:

Electromagnetism chooses μ^+, e^+, X^+, Z^+.
Strong interactions choose Z^+, X^+, X^o, Y^o.
Weak interactions choose $\mu_r^+ + Z_l^+$, $e_r^+ + X_l^+$, $\nu_r + X_l'$, $\nu_r' + Y_l'$,

* The mass formulae are

$$(\eta-\pi)(\chi-\pi) = \tfrac{2}{3}(K-\pi)(2\eta+2\chi-\pi-3K)$$
$$3S + \pi = 2(\eta + \chi)$$
$$S + \pi = D + K$$

The agreement between our predicted value of $m(\chi)$ = 950 MeV and the observed mass of $\eta\pi\pi$ of 959 MeV is very sensitive to the input masses. If we vary $m(K)$ by ± 2 MeV, we obtain $m(\chi)$ = 950 ± 50 MeV. A 12 MeV variation of $m(\eta)$ sends $m(\chi)$ to ∞. These mass formulae have been derived independently by M. Nauenberg [5].

** In a tadpole theory mass differences arise from nonvanishing expectation values of scalar meson fields. If the coupling of the scalar mesons φ to the vector mesons is gauge-invariant, $\frac{1}{4} f\varphi\, V_{\mu\nu} V^{\mu\nu}$, then the vector meson mass becomes $m_0^2(1 + f\langle\varphi\rangle)^{-1}$.

† It is also interesting that $\{W, W^+\} = 1$.

Table 2
Decays of charmed vector mesons *.

C=1 meson	Y	Channel	Strength	Estimated rate
$S^+(940)$	1	$\pi\pi$	$\cos^4\theta$	$\sim 10^{13}$-10^{14} sec^{-1}
		$K\pi$	$\cos^2\theta\sin^2\theta$	$\sim 10^{13}$-10^{14}
$D^+(770)$	0	$\pi\pi$	$\cos^2\theta\sin^2\theta$	$\sim 10^{13}$-10^{14}
		$K\pi$	$\sin^4\theta$	$\sim 10^{11}$-10^{12}
$D_1^0(770)$	0	$\pi\pi\pi$	$\cos^2\theta\sin^2\theta$	$\sim 10^{14}$
		$\bar{K}\pi$	$\cos^4\theta$	$\sim 10^{13}$-10^{14}
		$\pi\pi$	$\cos^2\theta\sin^2\theta$	$\sim 10^{13}$-10^{14}
$D_2^0(770)$	0	$\bar{K}\pi$	$\cos^4\theta$	$\sim 10^{13}$-10^{14}

* Computed on the basis that the mixing of the 15 vector mesons by the weak interactions is the dominant process.

Table 3
Decays of charmed pseudoscalar mesons.

C=1 meson	Y	Channel	Strength	Estimated rate
$S^+(900)$	1	$\eta\pi$	$\cos^4\theta$	$\sim 10^{11}$-10^{12} sec^{-1}
		$\pi\pi\pi$	$\cos^4\theta$	$\sim 10^{10}$
		$K\pi$	$\cos^2\theta\sin^2\theta$	$\sim 10^{10}$
		$\mu\nu$	$\cos^2\theta$	$\sim 10^{9}$-10^{10}
$D^+(760)$	0	$\bar{K}\pi$	$\cos^4\theta$	$\sim 10^{11}$-10^{12}
		$\eta\pi$	$\cos^2\theta\sin^2\theta$	$\sim 10^{10}$
		$\mu\nu$	$\sin^2\theta$	$\sim 10^{8}$-10^{9}
$D_1^0(760)$	0	$\bar{K}\pi$	$\cos^4\theta$	$\sim 10^{11}$-10^{12}
		$\pi\pi$	$\cos^2\theta\sin^2\theta$	$\sim 10^{10}$-10^{11}
		$\eta\pi$	$\cos^2\theta\sin^2\theta$	$\sim 10^{10}$
$D_2^0(760)$	0	$\bar{K}\pi$	$\cos^4\theta$	$\sim 10^{11}$-10^{12}

where r and l refer to chiralities. In terms of these fields weak interactions conserve "parity".

Selection rules for leptonic weak interactions follow from the form of the current. "Fast" leptonic decays have matrix elements proportional to $\cos\theta$ and satisfy $\Delta Y = \Delta C = 0$, $\Delta Q = \pm 1$, $\Delta I = 1$, or $\Delta Y = \Delta C = \Delta Q = \pm 1$, $\Delta I = 0$. "Slow" leptonic decays have matrix elements proportional to $\sin\theta$ and satisfy $\Delta Y = \Delta Q = \pm 1$, $\Delta C = 0$, $\Delta I = \frac{1}{2}$ or $\Delta C = \Delta Q = \pm 1$, $\Delta Y = 0$, $\Delta I = \frac{1}{2}$. The angle θ may be identified with the Cabbibo angle [9], but in this model the weak current transforms under SU(3) like the superposition of a member of a triplet and a member of an octet.

Nonleptonic interactions are of three kinds. "Fast" decays involve $\cos^2\theta$ and satisfy $\Delta Y = \Delta C = \pm 1$. "Slow" decays involve $\sin\theta\cos\theta$ and satisfy $\Delta Y = 0$, $\Delta C = \pm 1$, or $\Delta C = 0$, $\Delta Y = \pm 1$. "Very slow" decays involve $\sin^2\theta$ and satisfy $\Delta C = -\Delta Y = \pm 1$. There is no built-in-nonleptonic $\Delta I = \frac{1}{2}$ rule, and the familiar $\Delta C = 0$, $\Delta Y = \pm 1$ processes are suppressed by $\sin^2\theta$ in rate. The problem of the rapid rate of $\Delta I = \frac{1}{2}$ non-leptonic processes remains, and it appears to be necessary to invoke some mechanism (tadpoles [7]) for the selective enhancement of this channel [8,10] compared to $\Delta I = \frac{3}{2}$.

The model is vulnerable to rapid destruction by the experimentalists. The main prediction is the existence of the charmed $S^+_{p,v}$ and $D^{+,0}_{p,v}$ mesons which can be produced in pairs in π-p, $\bar{K}$-p and $\bar{p}$-p reactions, followed by weak but rapid decays into both Y-conserving and Y-violating channels. The baryon-lepton analogy lets us guess the order of magnitude of the decay rates, and although the numbers cannot be taken too seriously, we summarize them in tables 2 and 3.

Unless the charmed baryons have mass less than or the order of 2 GeV they decay strongly into the mesons. If they are a little lighter, they probably decay nonleptonically with rates $\gtrsim 10^{11}$-10^{12} sec^{-1}, and with branching ratios into leptonic modes of a few percent.

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