The negative proton: Its earliest history

Helge Kragh

Citation: American Journal of Physics **57**, 1034 (1989); doi: 10.1119/1.15815
View online: https://doi.org/10.1119/1.15815
View Table of Contents: http://aapt.scitation.org/toc/ajp/57/11
Published by the American Association of Physics Teachers

which are ordinarily not discussed at the undergraduate level. Students should be aware that its accuracy can depend on factors such as the size of the array and the number of iterations. Beginning exercises should involve a comparison of some results of the numerical method with the corresponding analytical solutions to get a feeling for the problems which can arise. The spreadsheet method is very flexible and can be extended to more advanced problems, which might be of interest for student research projects.

ACKNOWLEDGMENTS

I would like to express my thanks to Dr. Delbert Lessor of Batelle Pacific Northwest Laboratories and Dr. Bruce McLeod of Montana State University for their helpful suggestions. This work was supported by a contract from the Electric Power Research Institute.

[1]Richard Feinberg and Max Knittel, "Microcomputer spreadsheet programs in the physics laboratory," Am. J. Phys. **53**, 631–634 (1985).

[2]Francis X. Hart, "Software for introductory physics," Am. J. Phys. **55**, 200–201 (1987).

[3]Richard L. Bowman, "Electronic spreadsheets for physics students' use," Am. J. Phys. **56**, 184–185 (1988).

[4]Joseph M. Crowley, "Electrostatic fields on PC spreadsheets," J. Electrostat. **19**, 137–149 (1987).

[5]T. T. Crow, "Solutions to Laplace's equation using spreadsheets on a personal computer," Am. J. Phys. **55**, 817–823 (1987).

[6]Microsoft Corporation, 10700 Northrup Way, Box 97200, Bellevue, WA 98009.

[7]William T. Kaune and Murlin F. Gillis, "General properties of the interaction between animals and ELF electric fields," Bioelectromagnetics **2**, 1–11 (1981).

[8]W. T. Kaune and F. A. McCreary, "Numerical calculation and measurement of 60-Hz current densities induced in an upright grounded cylinder," Bioelectromagnetics **6**, 209–220 (1985).

[9]E. Isaacson and H. B. Keller, *Analysis of Numerical Methods* (Wiley, New York, 1966), pp. 467–470.

[10]G. E. Forsythe and W. R. Wasow, *Finite-Difference Methods for Partial Differential Equations* (Wiley, New York, 1960), pp. 242–283.

[11]D. Young, "The numerical solution of elliptic and parabolic partial differential equations," in *Survey of Numerical Analysis*, edited by J. Todd (McGraw-Hill, New York, 1962), pp. 398–407.

The negative proton: Its earliest history

Helge Kragh[a)]

Department of History, Cornell University, Ithaca, New York 14853

(Received 10 October 1988; accepted for publication 25 January 1989)

After the discovery of the positron in 1932, several physicists, including Bohr and Gamow, speculated that negative protons existed too. Other physicists considered antiprotons in the sense first suggested by Dirac in 1931. This article examines the early history of the concept of the negative proton and its application in nuclear and cosmic ray physics.

I. INTRODUCTION

> Alice laughed. "There's no use trying" she said: "One *can't* believe impossible things." "I daresay you haven't had much practice," said the Queen. "When I was your age I always did it for half-an-hour a day. Why, sometimes I've believed as many as six impossible things before breakfast." (G. Gamow, Nature **135**, 858 (1935), quoting Lewis Carroll's *Alice in Wonderland*).

The negative proton was detected experimentally in 1955 in a Nobel prize rewarded experiment by Emilio Segré and Owen Chamberlain. What they discovered, and what they earned their Nobel prize for, was the antiproton, usually identified with the negative proton. It is not well known, and is not revealed by either the Nobel lectures or other retrospective accounts, that the history of the negative proton dates back to the early 1930s, when the hypothetical particle was discussed by many physicists.[1] However, in a historical context, the negative proton should be distinguished from the antiproton: Although an antiproton is a negative proton, a negative proton is not necessarily an antiproton, and most references to the negative proton during the 1930s were in fact not to antiprotons. The present article examines the early status of the negative proton, primarily during the 1930s.

II. DIRAC'S ANTIPROTON

During the 1920s, the two-particle paradigm reigned supreme in atomic physics; according to this view, all matter consisted of two elementary particles, the negative electron and the positive proton. Speculations on other elementary particles figured rarely, and then mostly at the fringe of the physics literature. In 1922 Oliver Lodge, then 73 years old and well outside mainstream physics, suggested that positive electrons might exist inside the proton. Stimulated by Lodge's speculations, Horace Poole from Dublin introduced the idea of "light elements formed by the combination of positive and negative protons."[2] This is, to my knowledge, the first time the negative proton appears in physics.

Apart from Poole's speculative reference to the negative proton, the idea of an elementary particle with protonic mass but opposite charge was introduced by Paul Dirac in 1931. The year before he had introduced the concept of antiparticles of "holes," first believing that antielectrons were identical with protons.[3] In his classic paper of 1931, he realized that protons and electrons were unconnected and abandoned the "dream of philosophers," as he called the attempt to reduce all matter to manifestations of electrons. Instead, he suggested the existence of three new elementary particles: the magnetic monopole, the antielectron, and the antiproton. Dirac's negative protons, alias antiprotons, appeared as holes in a "sea" of unobservable negative-energy protons, and were briefly introduced as follows: "Presumably the protons will have their own negative-energy states, all of which normally are occupied, an unoccupied one appearing as an antiproton."[4]

Dirac's suggestion was further developed in August 1933 by the Rumanian–French physicist Jean Placinteanu. He assumed the linear Dirac equation to be valid also for protons, in which case the equation for the antiproton follows directly. A negative Dirac proton might be produced together with a positive proton by a photon of extreme energy. "One is thus led to admit the existence of the *negative proton*, symmetrical to the experimentally known proton exactly in the same way as the positive electron is symmetrical to the negative electron. Moreover, this *negative proton* has not been revealed experimentally, perhaps because of the very high energies which are necessary," he argued.[5] Placinteanu did not believe that the mass of the antiproton (or positron) was necessarily equal to that of the proton (or electron). In accordance with de Broglie's hypothesis of nonzero photon mass, he argued that a determination of the particle–antiparticle mass difference would yield the photon mass and then provide a test of de Broglie's idea.[6]

Very few physicists joined Placinteanu in discussing the negative proton as a Dirac particle. In Italy, Gleb Wataghin entertained similar ideas in a paper of 1935, in which he went a step further and introduced for the first time the antineutron.[7]

Following Dirac's 1931 remark, the next reference to antiprotons was that of the Russian physicist H. Mandel. In October 1932 he mentioned briefly that the existence of five elementary particles (proton, electron, antiproton, positron, and neutron) called for an attempt to reduce this unpleasantly large number.[8] In July of the following year, Reinhold Fürth at the German University in Prague discussed the subject more detailedly and speculatively. Fürth had for some years cultivated numerological speculation à la Eddington, suggesting, among other things, that a high-energy light quantum is really a proton–electron doublet, a neutron.[9] In his paper of 1933 he "deduced" the proton–electron mass ratio to be 1838.4 ± 1, and went on to consider matter made up of positive electrons and negative protons. A neutron, Fürth suggested, may under certain circumstances disintegrate into a positron and a negative proton, but then the latter particle would almost at once annihilate with an ordinary proton. However, "it appears perfectly possible that the relationship is just reversed in other parts of the universe, not belonging to our system of stars; in those parts there may only exist, in addition to neutrons, negative protons and positive electrons."[10] Fürth believed that the fact that the positron was discovered in the cosmic radiation indicated evidence of such antistars.

Dirac himself only returned publicly to the antiproton in December 1933, in his Nobel address. Referring to the recently measured magnetic moment of the proton, he mentioned that protons might perhaps not be described by the same theory as the electron and thus might not, after all, have an antiparticle. However, this reservation did not keep Dirac from ending his Nobel lecture in the following way, unusually speculative for him[11]:

> In any case I think it is probable that negative protons can exist ... If we accept the view of complete symmetry between positive and negative electric charge so far as concerns the fundamental laws of Nature, we must regard it rather as an accident that the Earth (and presumably the whole solar system), contains a preponderance of negative electrons and positive protons. It is quite possible that for some of the stars it is the other way about, these stars being built up mainly of positrons and negative protons. In fact, there may be half the stars of each kind. The two kinds of stars would both show exactly the same spectra, and there would be no way of distinguishing them by present astronomical methods.

Dirac did not pursue the idea of antiprotons or negative protons, but other physicists in the period occupied themselves with the question. The reference to negative protons in the Nobel address may have been inspired by his correspondence with Igor Tamm in Moscow. On 21 November 1933 Tamm wrote Dirac about his ideas of the composition of elementary particles, according to which a proton (π) consisted of a neutron (ν) and a positron (ϵ^+). "As to the question, why only positive protons ($\pi^+ = \nu + \epsilon^+$) and not the negative ones ($\pi^- = \nu + \epsilon^-$) are met with, I still hope, that the answer may be found in the difference of the direction of the magnetic moment relative to the direction of the spin in the case of an electron and in the case of a positron, this difference causing a large enough difference in the energy of the lowest state of the systems $\nu + \epsilon^+$ and $\nu + \epsilon^-$. But I have nothing new to say about it."[12] There seemed to be two places to look for negative protons, the still mysterious cosmic radiation and the equally mysterious atomic nucleus. Both avenues were followed.

III. THE NEGATIVE PROTON

The idea of negative protons as possible constituents of atomic nuclei was ventured quite frequently in the period 1933–36, in publications as well as in informal discussions. Such a suggestion was not unnatural at a time when the neutron, the positron, and the neutrino had just entered nuclear physics (Pauli at first believed the neutrino to reside in the nucleus). On 7 October 1933 Heisenberg wrote to Pauli that although he did not accept the idea of a composite neutron, he could well imagine the process $n \rightarrow p^- + e^+ + (\nu?)$, in addition to the more conventional disintegration $n \rightarrow p^+ + e^- + (\nu?)$.[13] Discussions in Copenhagen between Niels Bohr, George Gamow, and Evan Williams resulted in a suggestion of non-Diracian negative protons. On 20 April 1934, Bohr mentioned the suggestion to Heisenberg, who already had been informed by Pauli three days earlier[14]:

> Bohr thought much about negative protons and believes to have evidence for their existence in the cosmic radiation. There are theoretical as well as experimental (Stern's experiment) reasons for assuming that the relativistic Dirac wave equation is not at all applicable to heavy particles, and Bohr believes therefore that the negative protons *should not at all be related to the hole idea and hence not annihilate with the positive protons*!

Pauli first referred to the negative proton in September 1933 in a letter in which he argued that perhaps antineutrons exist too. Although not accepting the hole theory, he realized that Dirac's theory seemed to offer a new foundation for the problem of charge symmetry: "If it be true that the laws of nature are completely symmetrical with respect to positive and negative electricity (and all observable differences can be attributed to the *original state* of our environment)—if therefore a negative proton should exist—then the free neutron, assuming its magnetic moment is not zero, should be able to exist in *two* states: in one the magnetic moment would have the opposite direction of the angular momentum, in the other the same value but in the same direction."[15] Notice that the antineutron introduced here was not a Dirac particle. Contrary to the antineutron suggested by Wataghin in 1935, Pauli's particle would not annihilate with an ordinary neutron.

As emphasized by Pauli, Bohr believed that negative protons should be distinguished from antiprotons. This was a belief which had the full support of Pauli, who at the time was very critical of Dirac's hole theory.[16] In an unpublished manuscript from the spring of 1934, Bohr wrote that "we must expect that also particles with the same mass as the proton but with opposite charge will exist, and that such 'negative protons' will show a relation to ordinary protons which in essential respects will differ from the relationship between negative and positive electrons in Dirac's theory."[17] Bohr and his disciples relied on two arguments in their rejection of antiprotons. First, there was the problem of the magnetic moment. It was generally assumed that the proton, having spin one-half, would carry a magnetic moment equal to one nuclear magneton, a result which follows if it obeys the Dirac equation. It was therefore most surprising when Otto Stern and Otto Frisch in 1933 reported an experimental value of ca. 2.8 nuclear magnetons.[18] Second, Bohr made use of general arguments, based upon the principle of correspondence, according to which the domain of validity of Dirac's theory was limited to particles of a linear size small compared with the critical (Compton) length h/mc. While this is the case for the electron, the criterion is not fulfilled for the proton, the radius of which Bohr assumed was roughly equal to that of the classical electron. At the Solvay congress in October 1933 he argued that this implied that the concepts of Dirac's electron theory failed to make sense in the domain of nuclear phenomena.[19]

IV. NUCLEAR NEGATIVE PROTONS?

Bohr's view was reiterated in May 1934 by George Gamow in two papers on the negative proton. The Bohr–Gamow particle had nothing in common with Dirac's antiproton, but was argued mainly on simple symmetry considerations. From this position it followed that "there need be no annihilation when positive and negative protons are brought together."[20] Gamow realized that the suggestion of negative protons might seem speculative, a view he emphasized in his own way by introducing one of his papers on the subject with a quotation from *Alice in Wonderland*.[21] During the years 1934–36, he was quite confident that negative protons exist in the nucleus, a suggestion he published at least five times. In the conclusion of his 1935 paper, Gamow said that "there are so many indications of the existence of negative protons that the hope is justified that these as yet hypothetical particles, completing the symmetry of the physical world, will be found sooner or later."[22]

The indications referred to by Gamow were primarily the magnetic moments of the proton and the deuteron, nuclear theory, and certain features in radioactivity. He argued, among other things, that certain isomeric nuclei could be understood by assuming that one isomer contained a p^+p^- pair, the other an nn pair. Their different radioactive behavior would then be due to the processes $p^+ \to n + e^+$, $n \to p^+ + e^-$ and $p^- \to n + e^-$, $n \to p^- + e^+$, respectively. Further arguing that the exchange forces between negative and positive protons must be repulsive, Gamow concluded that presumably there was only a small number of negative protons in the nucleus. Granting this, the atomic mass number A and the atomic number Z would, of course, have to be reinterpreted. Instead of the ordinary relations

$$Z = N(p^+) \text{ and } A = N(p^+) + N(n),$$

one would have

$$Z = N(p^+) - N(p^-)$$

and

$$A = N(p^+) + N(n) + N(p^-).$$

As another result of the hypothesis of nuclear negative protons, Gamow mentioned that some of Fermi's recent nuclear experiments might involve expulsion of such particles. Such a process,

$${}^{A}_{Z}X + {}^{1}_{0}n \to {}^{A}_{Z+1}Y + {}^{\;\,1}_{-1}P,$$

seemed reasonable to Gamow because of the absence of a potential barrier.

Gamow was not alone in his advocacy of negative protons in the nucleus. In the summer of 1934 James Bartlett of Cambridge University drew attention to certain regularities in the distribution of isotopes among the elements. These, he argued, could be explained on the assumption of negative protons.[23] H. Schüler and T. Schmidt, two German astrophysicists, were, like many others in the period, concerned with explaining the magnetic moments of the deuteron and the proton. They suggested that the neutron should be conceived as a superposition of two states, one corresponding to $n = p^+e^-$ and the other to $n = p^-e^+$.[24] The idea of two neutrons, or neutron states, was independently suggested by S. Tolansky at Imperial College, London. Like most advocates of the negative proton, Tolansky relied essentially on a symmetry argument. "The confirmation of the existence of the positron suggests, on grounds of symmetry, that a negative proton might be expected to exist," he wrote.[25]

In September 1934 an informal conference, mainly on nuclear physics, was held in Copenhagen. Among the participants were Gamow, Bohr, Hevesy, Franck, Bethe, and Heisenberg. Negative protons were discussed, but there is no report to show to what extent. A month later a larger conference was held in London with contributions of Born, Gamow, Bethe, Fermi, Cockcroft, Blackett, and others. Gamow used the occasion to reiterate his belief in negative protons. One of his arguments was based on the so-called "beryllium anomaly," a reference to the puzzling stability of the Be-9 nucleus. The problem was this: According to mass spectrometric measurements by Kenneth Bainbridge, the mass of the Be-9 nucleus exceeded that of two α particles and one neutron (see Table I). Hence the spontaneous disintegration

Table I. Atomic mass data as accepted in 1934, 1937, and today. The 1934 data are from Oliphant's report to the London Conference (Ref. 27), the 1937 data from Bethe and Livingston's review article (Ref. 30). The Q value refers to the hypothetical disintegration of Be-9 into two alphas and one neutron.

	1934	1937	Present
n	1.0080	1.00813	1.00867
He-4	4.00216	4.00389	4.00260
Be-9	9.0155	9.01504	9.01219
Q (MeV)	+ 3.9	− 0.8	− 1.6

$$^9_4\mathrm{Be} \rightarrow ^4_2\mathrm{He} + ^4_2\mathrm{He} + ^1_0 n + 4\ \mathrm{MeV},$$

would be expected. However, although Caltech physicists R. M. Langer and R. W. Raitt claimed to have confirmed the activity, other measurements, performed by Lord Rayleigh and two groups of American physicists, failed to reveal any activity of Be-9.[26] By 1934 the stability of the isotope was an established fact. "We cannot find in this case any potential barrier of reasonable height to prevent the splitting of the nucleus,"[27] Gamow said at the London conference. As a "plausible explanation" he mentioned a hypothesis which earlier had been proposed by Pauli and Dirac, namely, that the Be-9 nucleus might consist of five protons, three neutrons, and one negative proton. A few months earlier, Gamow had reported the same idea to Bohr[28]:

> We [Gamow and Cockcroft] have been also discussing in which nuclear reaction a negative proton is likely to be emitted, he is going to try some of them. By the way, what would you say about the following explanation of recent Fermi results;
>
> $_{92}\mathrm{U} + {_0 n} \rightarrow {_{93}\mathrm{X}} + \bar{p}$?
>
> I spoke with Dirac about the stability of Berilium and he proposed the same point of view as Pauli in the discussion in Copenhagen: Be is stable because there is not enough neutrons to form two α-particles ($:^9_4\mathrm{Be} = 5p^+ + 1p^- + 3n$).

This solution to the beryllium anomaly was presumably first proposed by Pauli during a visit to Copenhagen in April 1934, at a time when Gamow stayed at Bohr's institute. On 11 June the same year, Gamow gave, in Dirac's presence, a talk on negative protons to the Kapitza Club in Cambridge, which must have been the occasion to which Gamow referred.[29]

The beryllium anomaly was soon resolved, not by introducing negative protons in the nucleus, but simply by improved experimental data. Hans Bethe and other physicists analyzed existing methods of mass determination and argued that nuclear masses should primarily be based on the Q values of nuclear reactions.[30] The new nuclear masses, as accepted in 1936–37, showed no disagreement with the stability of the Be-9 nucleus.

Returning to the London conference in 1934, Gamow further explained that if there existed a negative proton in the beryllium nucleus, one might anticipate the process

$$p^+ + p^- \rightarrow n + n,$$

yielding

$$(5p^+, 1p^-, 3n) \rightarrow (4p^+, 5n) \rightarrow \alpha + \alpha + n.$$

However, he argued that "the probability of such simultaneous transformations of two heavy particles must be expected to be very small and the Be-nucleus may exist for a fairly long time. From this point of view the life period of beryllium depends on the probability of double β-transformation of two heavy particles."[31] In the discussion following Gamow's talk, Fermi commented on the negative proton hypothesis, apparently supporting it: "I heartily agree with Dr. Gamow, that both his empirical evidence and the more general argument of symmetry strongly support the possibility of the existence of a negative proton."[32] In spite of these words, Fermi's support was at best ambiguous. He pointed out that the negative proton was not easily reconcilable with Heisenberg's theory of proton–neutron interaction; he also found it difficult to understand why there could only be a very small number of negative protons, the particles yet being present in nuclei as different as beryllium and lead.

The lack of experimental support for negative protons did not discourage Gamow. Experiments carried out at the Carnegie Institution by Merle Tuve and his group failed to reveal the particle, but as Gamow reported to Bohr, "I still hope that negative protons will be turned out sooner or later."[33] Still in 1936 he maintained his belief. In the second edition of his *Structure of Atomic Nuclei and Nuclear Transformation*, published in 1937, he spent four pages on the subject. He reported no new evidence, but merely reiterated earlier arguments, including his belief that negative protons and antiprotons were completely different things.[34]

V. COSMIC RAY NEGATIVE PROTONS?

Negative protons as a possible constituent of the cosmic radiation were first mentioned by Carl D. Anderson in his famous report of February 1933, in which the "positron" was announced. As a possibility for explaining certain cloud chamber tracks, Anderson mentioned that a primary cosmic ray might cause a nuclear neutron to disintegrate according to either $n \rightarrow e^- + p^+$ or $n \rightarrow e^+ + p^-$. "This [latter] alternative, however, postulates the existence in the nucleus of a proton of negative charge, no evidence for which exists."[35] Anderson shared the view of most physicists at the time, that the neutron is a proton–electron composite and not an elementary particle. Also he did not at the time identify the positron with Dirac's antielectron, and consequently the remark about negative protons did not refer to antiprotons either. In spite of his rejection of negative protons as candidates for cloud chamber events, Anderson argued that general symmetry reasons "should prove a stimulus to search for evidence of the existence of negative protons."[36]

The idea dismissed by Anderson received some support in connection with the puzzling absorption data of cosmic rays. In the spring of 1934 Evan Williams of Manchester University stayed in Copenhagen with Bohr and Gamow, trying to make sense of the ionization values recently obtained by Paul Kunze and others. In accordance with the views of Bohr and Gamow, he suggested that negative protons exist and that these, together with ordinary protons, make up the highly ionizing part of cosmic radiation.[37] Williams continued for some time to maintain this view. For example, he reiterated it at a conference held at the

Polytechnic University of Zürich (ETH) in the summer of 1936.[38]

The hypothesis of cosmic radiation negative protons was never common. But for lack of better candidates it was occasionally forwarded in order to explain the absorption data. Thus Homi Bhabha accepted Williams's view as the most reasonable way to explain coincidence-counter experiments carried out in 1934 by Bruno Rossi and S. Benedetti. In March 1937, a few months before the meson made its impact in Western physics and changed the entire basis of discussion, Bhabha concluded that "we are compelled to admit the existence of negative protons or new negative particles in appreciable numbers in cosmic radiation."[39]

The first claim of actually having found the negative proton was announced in December 1937. "The cosmic ray observations of Dr. Millikan and his collaborators provide the first experimental proof [of negative protons] sought by scientists all over the world," reported *The New York Times*, grossly exaggerating physicists' interest in the hypothetical particle.[40] The research referred to was conducted by Caltech physicists Robert Millikan, Carl Anderson, and Seth Neddermeyer, but only Millikan spoke for the group and wrote the report. Actually he was not certain if their data proved the existence of the negative proton, and wisely phrased the discovery claim conditionally: "They [the penetrating particles] must be either protons or else particles of intermediate mass between electrons and protons. If they are protons, since they are positive and negative in sign, a negative proton has been discovered."[41] At that time, it should be recalled, Anderson and Neddermeyer had already announced their discovery of their *mesotron*, later to be identified with the muon.

The mesotron or meson received general acceptance by 1938, at which time the cosmic radiation as well as the atomic nucleus were much better understood than they were a few years earlier. Interest in the negative proton declined, but did not vanish. During the next decade, suggestions continued, the hypothetical particle being associated with meson theory, cosmic radiation, or nuclear reactions.[42] In most of these suggestions the negative proton was not identified with the antiproton.

VI. CONCLUSION

The hypothetical negative proton, a particle which historically should be distinguished from the antiproton, received considerable attention during the 1930s. Its existence was suggested by prominent physicists, including Bohr, Gamow, and Williams, who saw in it a possibility of explaining some of the experimental puzzles which at the time plagued physics. Although only the antiproton could be justified theoretically, there was in the period more interest in the theoretically unjustified negative proton than in Dirac's particle. Of course, when the negative proton was finally discovered in 1955, it turned out to be, after all, an antiproton.

ACKNOWLEDGMENT

An early version of the present article was delivered at the 74th Congress of the Italian Physical Society, held in Urbino in October 1988. I gratefully acknowledge the invitation from the Italian Physical Society and support from the University of Parma, Italy.

[a] Permanent address: Magnolievangen 41, 3450 Allerød, Denmark.

[1] See the Nobel lectures by E. Segré and O. Chamberlain, in *Nobel Lectures. Physics 1942-1962* (Elsevier, Amsterdam, 1964), pp. 489–520, and E. Segré, "Antinucleons," Am. J. Phys. **25**, 363–369 (1957). Neither do negative protons figure in Abrahams Pais's comprehensive account of particle history, *Inward Bound* (Clarendon, Oxford, 1986).

[2] O. Lodge, "Speculations concerning the positive electron," Nature **110**, 696 (1922); Horace Poole, "Speculations concerning the positive electron," Nature **111**, 15 (1923).

[3] P. A. M. Dirac, "A theory of electrons and protons," Proc. R. Soc. London Ser. A **126**, 360–365 (1930). See also D. F. Moyer, "Vindication of Dirac's electron," Am. J. Phys. **49**, 1120–1124 (1981), and Helge Kragh, *Dirac. A Scientific Biography* [Cambridge U. P., (to be published)], Chap. 5.

[4] P. A. M. Dirac, "Quantised singularities in the electromagnetic field," Proc. R. Soc. London Ser. A **133**, 60–72 (1931), on p. 62.

[5] J. J. Placinteanu, "Considérations théorique sur la constitution des neutrons, électrons positifs et photons. Existence des protons négatifs," C. R. Acad. Sci. **197**, 549–552 (1933), on p. 550.

[6] De Broglie attempted to build up a theory of light in which photons were pairs of neutrinos-antineutrinos. See L. de Broglie, "Sur la nature du photon," C. R. Acad. Sci. **198**, 135–138 (1934).

[7] G. Wataghin, "Sulla teoria dei protoni e neutroni," Rend. R. Accad. Naz. Lincei **21**, 703–708 (1935).

[8] H. Mandel, "Einige überlegungen über die mögliche Beschaffenheit des Neutrons," Phys. Z. Sowjetunion **2**, 286–290 (1932).

[9] R. Fürth, "über die Massen von Proton und Elektron," Naturwissenschaften **17**, 728–729 (1929).

[10] R. Fürth, "Einige Bemerkungen zum Problem der Neutronen und positiven Elektronen," Z. Phys. **85**, 294–299 (1933), on p. 298.

[11] P. A. M. Dirac, "Theory of electrons and positrons," in *Nobel Lectures, Physics 1922–41* (Elsevier, Amsterdam, 1963), pp. 320–325, on p. 325. Nobel lecture of 12 December 1933.

[12] Tamm to Dirac, 21 November 1933 (Tamm–Dirac correspondence).

[13] Heisenberg to Pauli, 7 October 1933, in *Wissenschaftlicher Briefwechsel*, edited by K. v. Meyenn, A. Hermann, and V. F. Weisskopf (Springer, Berlin, 1985), Vol. 2, p. 218. Also Heisenberg to Pauli, 7 November 1934. The symbol (ν?) refers to a possible neutrino.

[14] Pauli to Heisenberg, 17 April 1934, Ref. 13, p. 316. Bohr to Heisenberg, 20 April 1934, in *Niels Bohr. Collected Works*, edited by E. Rüdinger (North-Holland, Amsterdam, 1986), Vol. 9, p. 579. The penetrating negative cosmic ray particles turned out to be muons and not negative protons. For the unraveling of the particle content of the cosmic radiation, see Peter Galison, *How Experiments End* (University of Chicago Press, Chicago, 1987), pp. 75–134.

[15] Pauli to Heisenberg, 29 September 1933, Ref. 13, p. 213.

[16] Pauli to Heisenberg, 17 April 1934.

[17] "The electron and proton," Ref. 14, Vol. 5, p. 124. Undated, but most likely from spring 1934.

[18] O. R. Frisch and O. Stern, "über die magnetische Ablenkung von Wasserstoffmolekülen und das magnetische Moment des Protons," Z. Phys. **85**, 4–16 (1933).

[19] N. Bohr in discussion following Dirac's 1933 Solvay lecture, in *Structure et Propriétés des Noyaux Atomiques* (Gauthiers-Villars, Paris, 1934), p. 216.

[20] G. Gamow, "Negative protons and nuclear structure," Phys. Rev. **45**, 728–729 (1934), on p. 728. Also G. Gamow, "über den heutigen Stand (20 Mai 1934) der Theorie des β-Zerfalls," Phys. Z. **35**, 533–542 (1934).

[21] G. Gamow, "The negative proton," Nature **135**, 858–861 (1935). Interview with Gamow, conducted by C. Weiner, 25 April 1968 (AIP).

[22] Reference 21, p. 861.

[23] James H. Bartlett, "Negative protons in the nucleus?," Phys. Rev. **46**, 435 (1934).

[24] H. Schüler and T. Schmidt, "Eine Bemerkung zur Theorie des Neutrons," Naturwissenschaften **22**, 418 (1934).

[25] S. Tolansky, "Negative nuclear spins and a proposed negative proton," Nature **134**, 26 (1934).

[26] R. M. Langer and R. W. Raitt, "A new kind of radioactivity," Phys. Rev. **43**, 585 (1933); (Lord) Rayleigh, "Beryllium and helium," Nature **131**, 724 (1933); Robley D. Evans and M. C. Henderson, "Failure

to detect radioactivity in beryllium with the linear amplifier," Phys. Rev. 44, 59 (1933); David M. Gans, William D. Harkins, and Henry W. Newton, "Failure to detect radioactivity of beryllium with the Wilson cloud chamber," Phys. Rev. 44, 310 (1933).

[27]George Gamow, "General stability-problems of atomic nuclei," in *International Conference on Physics*, Vol. 1, *Nuclear Physics*, edited by J. H. Awberry (Cambridge U. P., Cambridge, 1935), pp. 60–71, on p. 62.

[28]Gamow to Bohr, 13 June 1934 (Archive for History of Quantum Physics). Published references to the Pauli–Dirac conjecture include Gamow, Ref. 27, p. 62, and E. Gapon and D. Iwanenko, "Alpha particles in light nuclei," C. R. Acad. Sci. URSS 4, 276–277 (1934).

[29]G. Gamow, talk on "Negative protons," Cambridge University, 11 June 1934 (Kapitza Club Minute Book, AHQP).

[30]H. A. Bethe, "Masses of light atoms from transmutation data," Phys. Rev. 47, 633–634 (1935); and in more details in M. Stanley Livingston and H. A. Bethe, "Nuclear dynamics, experimental," Rev. Mod. Phys. 9, 245–390 (1937) [reprinted in *Basic Bethe. Seminal Articles on Nuclear Physics 1936–37* (AIP, New York, 1986), pp. 331–476].

[31]Gamow, Ref. 27, p. 62.

[32]Reference 27, p. 68.

[33]Gamow to Bohr, 20 January 1935 (AHQP).

[34]G. Gamow, *Structure of Atomic Nuclei and Nuclear Transformation* (Clarendon, Oxford, 1937), pp. 14–18. The first edition of 1931 was entitled *Constitution of Atomic Nuclei and Radioactivity.*

[35]C. D. Anderson, "The positive electron," Phys. Rev. 43, 491–494 (1933), on p. 494.

[36]Reference 35.

[37]E. J. Williams, "Nature of the high energy particles of penetrating radiation and status of ionization and radiation formulae," Phys. Rev. 45, 729–730 (1934).

[38]E. J. Williams, "General survey of theory and experiment for high-energy electrons," in *Kernphysik*, edited by E. Bretscher (Springer, Berlin, 1936), pp. 123–141, on p. 123.

[39]H. J. Bhabha, "Negative protons in the cosmic radiation," Nature 139, 415–416 (1937).

[40]The New York Times, 12 December 1937. Millikan's other discovery claim was of an "X-particle" of mass intermediate between the electron and the proton.

[41]R. A. Millikan, Carnegie Institution Yearbook, No. 36, 1937, p. 364.

[42]See the references in E. Broda, N. Feather, and D. Wilkinson, "A search for negative protons emitted as a result of fission," in *Report of an International Conference on Fundamental Particles and Low Temperatures* (Taylor and Francis, London, 1947), Vol. 1, pp. 114–124.

Some results on the relativistic Doppler effect for accelerated motion

W. Cochran
Department of Physics, The University, Edinburgh EH9 3JZ, United Kingdom

(Received 27 April 1988; accepted for publication 25 January 1989)

It is shown how the Doppler-shifted frequencies that result from the uniform acceleration of a source of light, and/or of the receiver, can be derived, and that very simple formulas often apply. For completeness it is shown how a basic equation can be extended to apply in an accelerated frame of reference.

I. INTRODUCTION

Textbooks on special relativity usually consider the Doppler effect for light only for the situation where source and receiver are in uniform relative motion, although Rindler[1] explains how the standard formulas are to be interpreted when the motion is not uniform. A generalization of these formulas (for example, from $[(1-u/c)/(1+u/c)]^{1/2}$ for the frequency ratio for uniform relative motion in one dimension) was recently published by Bachman.[2] A succinct derivation is given below. The frequency ratio for accelerated motion can in principle be worked out in any convenient frame of reference. In practice, a calculation will always be simplest when based in an inertial frame, but as far as I am aware an explicit result has been obtained for one situation only, by Hamilton,[3] in a study of the properties of light in an accelerated frame. The topic is in fact within the scope of an undergraduate course on special relativity.

Let x_s and x_r be the positions of source and receiver at times t_s and t_r, respectively, in an inertial frame—their motion is, meantime, in one dimension. Successive light flashes are emitted at t_s and $t_s + dt_s$, and received at t_r and $t_r + dt_r$, respectively. The corresponding intervals of proper time are $d\tau_s$ and $d\tau_r$. Then

$$\frac{\nu_r}{\nu_s} = \frac{d\tau_s}{d\tau_r} \tag{1}$$

$$= \frac{dt_s(1-u_s^2/c^2)^{1/2}}{dt_r(1-u_r^2/c^2)^{1/2}}, \tag{2}$$

with $x_r > x_s$, $c(t_r - t_s) = x_r - x_s$, and therefore

$$dt_r(1-u_r/c) = dt_s(1-u_s/c). \tag{3}$$

Combining these, we have the required result:

$$\frac{\nu_r}{\nu_s} = \left(\frac{1+u_s/c}{1-u_s/c}\right)^{1/2}\left(\frac{1-u_r/c}{1+u_r/c}\right)^{1/2}. \tag{4}$$

When u_s and u_r are constant, Eq. (4) reduces to the standard result, that

$$\frac{\nu_r}{\nu_s} = \left(\frac{1-u/c}{1+u/c}\right)^{1/2},$$

involving the relative velocity $u = (u_r - u_s)/(1 - u_r u_s/c^2)$, but as Bachman has emphasized, u_s and u_r must otherwise be taken at different times.

We assume that the source is controlled by an ideal clock, the mechanism of which is unaffected by acceleration, so that ν_s is a constant. It follows from the form of Eq. (1) that ν_r/ν_s is invariant in a transformation to another

