Week 10: Nuclear Reactions (Undergraduate)

Nuclear reaction experiments fire projectiles at target nuclei in order to alter them somehow (the altered nuclei, which are not usually directly observed, are referred to as residual nuclei) while emitting single particles (which are observed):

$$\text{Projectile} + \text{Target Nucleus} \rightarrow \text{Residual Nucleus} + \text{Detected Particle}$$

Nuclear physicists condense this reaction equation (for some reason) as:

$$\text{Target Nucleus(Projectile, Detected Particle)Residual Nucleus}$$

which symbolically can be written,

$$\text{P(p, d)D}$$

where P is the target nucleus (parent), p is the projectile, d is the detected particle, and D is the residual nucleus (daughter).

Note that in nuclear reactions, the total charge and the total number of nucleons after the interactions must be the same as the total charge and total number of nucleons before the interaction (the total charge and the total number of nucleons are conserved).

Typical projectiles and detected particles in nuclear reaction experiments include neutrons (n), protons (p or ${}^{1}_{1}$H), deuterons (d or ${}^{2}_{1}$H), tritons (t or ${}^{3}_{1}$H), helium-3 (h or ${}^{3}_{2}$He), helium-4 (α or alpha particles or ${}^{4}_{2}$He), electrons, and photons. Such experiments led to some of the earliest discoveries in sub-atomic physics. They continue today.

Rutherford, in 1919, bombarded nitrogen with an alpha particle and a proton was emitted. In 1925, Blackett identified the reaction as

$${}^{14}_{7}\text{N}(\alpha,\ p){}^{17}_{8}\text{O}$$

nitrogen being transformed into oxygen, the first example of artificial transmutation and the first realization that high energy collisions could increase the masses of nuclei as well as decrease them.

A reaction in which the projectile and detected particle are the same is classified as a scattering reaction. In this case, if the residual nucleus is left in its ground (lowest energy) state, the scattering reaction is classified as elastic; if the residual nucleus is left in an excited state, the scattering reaction is classified as inelastic.

A reaction in which the projectile gains nucleons from the target nucleus is classified as a pickup reaction, for example,

$$\begin{gathered} {}^{16}_{8}\text{O}(d,\ t){}^{15}_{8}\text{O} \\ {}^{41}_{20}\text{Ca}(h,\ \alpha){}^{40}_{20}\text{Ca} \end{gathered}$$

A reaction in which the projectile loses nucleons to the target nucleus is classified as a stripping reaction, for example,

$$^{90}_{40}\text{Zr}(d,\ p)^{91}_{40}\text{Zr}$$
$$^{23}_{11}\text{Na}(h,\ d)^{24}_{12}\text{Mg}$$

Pickup and stripping reactions are often direct: the nucleon(s) involved enter(s) or leave(s) its(their) shell(s) in the target nucleus without affecting other nucleons. Such direct reactions are the typical outcome with high energy projectiles.

At lower projectile energies, projectile and target can form a compound nucleus, which exists for $\sim 10^{-16}$ s before transforming. This is long enough to assume the subsequent transformation of the compound nucleus does not depend on how it was formed.[1] It is not unusual for a number of different reactions to produce the same compound nucleus which subsequently transforms with characteristic probabilities in a number of different ways (modes or channels). For example,

$$\left.\begin{array}{l} ^{19}_{9}\text{F} + p \\ ^{17}_{8}\text{O} + h \\ ^{16}_{8}\text{O} + \alpha \\ ^{14}_{7}\text{N} +^{6}_{3}\text{Li} \\ ^{12}_{6}\text{C} +^{8}_{4}\text{Be} \\ ^{10}_{5}\text{B} +^{10}_{5}\text{B} \end{array}\right\} \rightarrow [^{20}_{10}\text{Ne}]^* \rightarrow \left\{\begin{array}{l} ^{19}_{9}\text{F} + p \\ ^{19}_{10}\text{Ne} + n \\ ^{20}_{10}\text{Ne} + \gamma \\ ^{18}_{9}\text{F} + d \\ ^{17}_{9}\text{F} + t \\ ^{17}_{8}\text{O} + h \\ ^{16}_{8}\text{O} + \alpha \\ ^{14}_{7}\text{N} +^{6}_{3}\text{Li} \\ ^{13}_{7}\text{N} +^{7}_{3}\text{Li} \\ ^{12}_{6}\text{C} +^{8}_{4}\text{Be} \\ ^{11}_{6}\text{C} +^{9}_{4}\text{Be} \\ ^{10}_{5}\text{B} +^{10}_{5}\text{B} \\ ^{9}_{5}\text{B} +^{11}_{5}\text{B} \end{array}\right.$$

1. **What particle does X represent in the following reactions:**
   (a) $^{18}_{8}\mathbf{O}(d,\ p)X$
   (b) $X(p,\ \alpha)^{87}_{39}\mathbf{Y}$
   (c) $^{122}_{52}\mathbf{Te}(X,\ d)^{124}_{53}\mathbf{I}$
   (d) $^{182}_{74}\mathbf{W}(^{3}_{2}\mathbf{He},\ n)X$
   (e) $^{42}_{20}\mathbf{Ca}(^{6}_{3}\mathbf{Li},\ X)^{45}_{21}\mathbf{Sc}$
2. **An α-particle interacts with a $^{19}_{9}$F nucleus and forms a compound nucleus. What would be that compound nucleus and what might be some of its final reaction modes?**

Nuclear reaction experiments, particularly those focused on the energetics of the reaction rather than (differential) cross-sections, are frequently analyzed in the center-of-mass (CM) frame of the reaction. The CM frame moves with

[1] As short as 10^{-16} s might seem, it's 100,000 times longer than the time it takes a projectile to traverse a nuclear distance, $\sim 10^{-14}$ m / 10^{7} m/s $= 10^{-21}$ s.

constant velocity in the laboratory (Lab) frame, and the total momentum of the system is zero in the CM frame.

The initial system can be considered independent of the final system in the CM. In the initial system, if the target is at rest in the laboratory frame, the center of mass moves in the direction of the projectile in the Lab frame. In the final system, the direction of the scattered projectile can be chosen as the direction of the center of mass in the laboratory frame.

Labelling the magnitude of the projectile's velocity v_P, of the target's velocity v_T, and of the center of mass v_{CM}, then

$$v'_P = v_P - v_{\mathrm{CM}}$$
$$v'_T = v_T - v_{\mathrm{CM}}$$

are the projectile and target velocities in the CM. Taking the target to be at rest in the laboratory frame ($v_T = 0$),

$$v'_T = -v_{\mathrm{CM}}$$

Since nothing external affects the reaction, the total momentum of the system is conserved.

$$Mv_{\mathrm{CM}} = (m_P + m_T)v_{\mathrm{CM}} = m_P v_P + m_T v_T$$

or

$$v_{\mathrm{CM}} = \frac{m_P v_P + m_T v_T}{m_P + m_T} = \frac{m_P v_P}{m_P + m_T}$$

if the target is at rest in the laboratory frame, in which case,

$$v'_P = \frac{m_T}{m_P + m_T}(v_P - v_T) = \frac{m_T}{m_P + m_T} v_P$$
$$v'_T = \frac{m_P}{m_P + m_T}(v_T - v_P) = -\frac{m_P}{m_P + m_T} v_P$$

before the reaction. After the reaction, the scattered projectile and target (assuming these are the only two reaction products) separate with equal, but opposite, momenta in the CM.

Nuclear reactions can absorb or release energy, increasing or decreasing the total kinetic energy. The energy change is referred to as the Q-value:

$$Q \equiv K_{\mathrm{after}} - K_{\mathrm{before}}$$

Since $E = M + K$ is conserved, where M is the mass of the system,

$$Q = M_{\mathrm{before}} - M_{\mathrm{after}}$$

If $Q > 0$, the reaction is called exothermic or exoergic, and the reaction can proceed even if both initial particles are at rest. $Q < 0$ reactions are called endothermic or endoergic, and will proceed only if the projectile's kinetic energy is above a certain threshold. If $Q = 0$, and the initial and final particles are the same, the reaction is elastic. In a general nuclear reaction,

$$Q = (M_P + m_p) - (m_d + M_D)$$

For a scattering reaction,

$$Q = M_P - M_D$$

since the projectile and detected particle are the same.

3. **A 6-MeV proton beam is directed at a stationary $^{12}_{6}$C target. What is the system's center-of-mass velocity (in m/s)? [Set $m_{\text{proton}} = 1$ u.]**

4. **Calculate the Q-values in natural units for**

   (a) $^{16}_{8}\mathrm{O}(\gamma,\ p)^{15}_{7}\mathrm{N}$

   (b) $^{42}_{20}\mathrm{Ca}(p,\ d)^{41}_{20}\mathrm{Ca}$

   (c) $^{150}_{62}\mathrm{Sm}(p,\ \alpha)^{147}_{61}\mathrm{Pm}$

5. **Discovery of the neutron:**

   (a) **The alpha-transition of $^{210}_{84}$Po produces 5.30-MeV α-particles. A beam of these α-particles impinging on a stationary $^{9}_{4}$Be target produces a neutral (uncharged) radiation. Assuming the reaction to be $^{9}_{4}\mathrm{Be}(\alpha,\ \gamma)^{13}_{6}\mathrm{C}$, calculate the energy of the emitted γ. [Assume a head-on collision between an α-particle and a $^{9}_{4}$Be nucleus at rest, resulting in a γ and a $^{13}_{6}$C nucleus moving in the same direction as the α-particle projectile. Use mass-energy conservation, but treat the momenta and energies of the particles with mass non-relativistically.]**

   (b) **The neutral radiation from the previous reaction aimed at a proton-rich paraffin target produces protons with kinetic energies as high as 5.7 MeV. When aimed at a $^{14}_{7}$N target, nitrogen nuclei recoil with kinetic energies as high as 1.4 MeV. What is the minimum photon energy that can produce these two results? Compare your answers to that of the previous calculation. [Hints: This is another sort of Compton scattering, but can be analyzed without special relativity, because the kinetic energies are much smaller than the target masses. Invoke energy and momentum conservation to find two equations for the two unknowns. Minimum photon energy corresponds to head-on collisions.]**

(c) Assume that the neutral radiation is a massive particle rather than a photon. With the kinetic energies of the protons and $^{14}_{7}$N nuclei of the previous problem, calculate the mass and initial kinetic energy of the massive particle.

(d) If the initial reaction is $^{9}_{4}\text{Be}(\alpha,\ n)^{12}_{6}\text{C}$ instead of $^{9}_{4}\text{Be}(\alpha,\ \gamma)^{13}_{6}\text{C}$, calculate the kinetic energy of the neutron and compare it to the kinetic energy found in the previous problem. [Because the inital and final masses are not the same, mass difference will contribute to the kinetic energies of the final state particles. Othewise, everything can be treated classically, since all kinetic energy values are (much) smaller than masses.]