Week 11: Nuclear Fission and Fusion (Undergraduate)

In nuclear fission, a nucleus with $A > 230$ absorbs a neutron and forms a compound nucleus which then splits into two medium-mass nuclear fragments and emits some number of neutrons. For example,

$$^{235}_{92}\mathrm{U} + ^{1}_{0}\mathrm{n} \rightarrow [^{236}_{92}\mathrm{U}]^{*} \rightarrow ^{A_1}_{Z_1}\mathrm{X} + ^{A_2}_{Z_2}\mathrm{Y} + \varepsilon ^{1}_{0}\mathrm{n}$$

where ε is an integer, $A_1 + A_2 + \varepsilon = 236$, and $Z_1 + Z_2 = 92$. On average, $M_1/M_2 \approx 3/2$, while ε depends on which fragments are produced. In the example, $\bar{\varepsilon} \approx 2.44$. Both fragments approximately retain the Z/N ratio of the parent and so lie below the stability curve in the neutron-rich region. They therefore tend to undergo negative beta decay, perhaps repeatedly, until reaching stability. Roughly 200 MeV of energy is released per fission (about 100 times more energy than the single-detected particles and residual nuclei reactions discussed previously). Of that 200 MeV, about 170 MeV goes to the kinetic energy of the fission fragments, about 5 MeV total goes to the neutrons, about 15 MeV goes to the negative beta particle (electron) and γ-rays released by the excited fragments, and about 10 MeV goes to the neutrinos of the beta-decays.

Compound nuclei are most likely to fission when formed with low-energy neutrons of $E \approx 0.04$ eV (referred to, at this low energy, as a thermal neutron). Slowing down the approximately 2 MeV neutrons emitted in the fission reaction to approximately 0.04 eV in order to induce additional fission reactions is a problem for nuclear engineering. Since, on average, more than one neutron is emitted, controlled fission requires absorbing excess neutrons–another problem for nuclear engineering. Uncontrolled reactions (a nuclear bomb) grow geometrically, and all the generated energy is released in a short time.

1. **What is the final kinetic energy of a 2-MeV neutron that collides elastically head-on with a quasi-free (loosely bound) proton at rest?**

2. **Few elastic neutron-proton collisions are head-on. On average, a neutron colliding with a quasi-free proton loses about half its energy. Approximately how many collisions are necessary to reduce a 2-MeV neutron to a thermal neutron (0.04 eV)?**

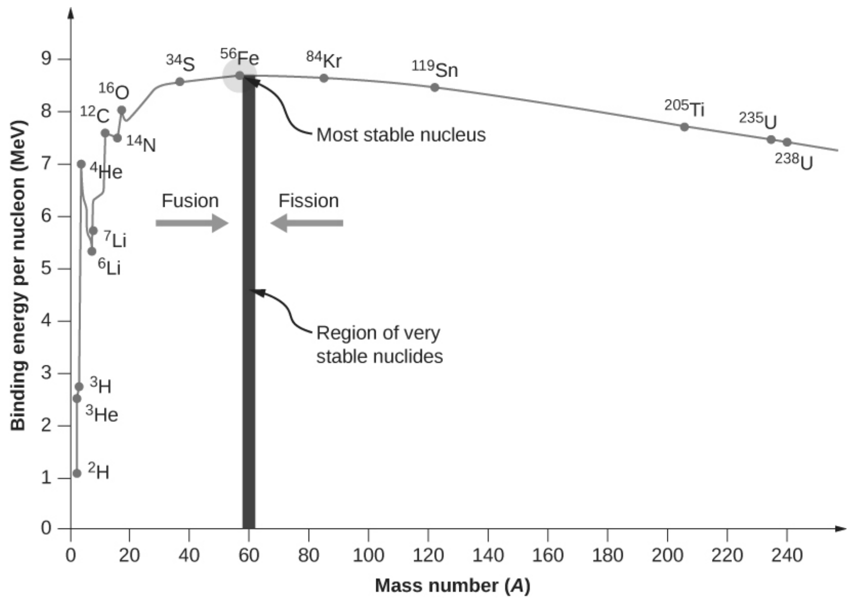
3. **A typical $^{235}_{92}\mathrm{U}$ fission reaction is**

$$^{235}_{92}\mathrm{U} + ^{1}_{0}\mathrm{n} \rightarrow [^{236}_{92}\mathrm{U}]^{*} \rightarrow ^{143}_{56}\mathrm{Ba} + ^{90}_{36}\mathrm{Kr} + 3^{1}_{0}\mathrm{n}$$

$^{143}_{56}\mathrm{Ba}$ and $^{90}_{36}\mathrm{Kr}$ then undergo series of beta decays before stable nuclei are reached.

(a) **Describe the two series and write out the final total reaction (that is, in terms of the final states, including all stable nuclei and decay products).**

(b) **Calculate the total kinetic energy of the final particles. This is the energy released in the fission reaction. Assume the neutrinos are massless.**



The figure shows the binding energy per nucleon as a function of mass number. Recall that the greater the binding energy, the more stable the object. Notice that $^{56}_{26}$Fe has the greatest binding energy per nucleon and is the most stable nucleus. For A larger than 60 or so, fission becomes a possibility. For A smaller than 56, however, stability is increased by adding nucleons. The change per nucleon is especially dramatic among the smallest nuclei until $^{4}_{2}$He, and then again from $^{6}_{3}$Li to $^{20}_{10}$Ne and other isobars (nuclei of equal mass number) of $A \approx 20$. Combining or fusing among these light nuclei to form more massive nuclei is not uncommon, as it leads to larger binding energy per nucleon. The mass lost goes to the release of energy, typically in the form of energetic photons and neutrinos as the fusion products settle into lower energy states.

For example, deuteron is formed when a proton and a neutron fuse,

$$^1_1\mathrm{H} + ^1_0\mathrm{n} \to ^2_1\mathrm{H}$$

a reaction in which $Q = 2.23$ MeV of energy is released. Two deuterons can then fuse to form an alpha-particle:

$$^2_1\mathrm{H} + ^2_1\mathrm{H} \to ^4_2\mathrm{He}$$

releasing $Q = 23.8$ MeV of energy.

While fission reactions release anywhere from 10 to 100 times more energy, fusion releases more energy per unit mass, given the smaller masses of the reactants.

These two examples are simplifications of the so-called proton-proton cycle that powers the sun and other "low temperature" stars.

$$
\begin{aligned}
{}_1^1\mathrm{H} + {}_1^1\mathrm{H} &\rightarrow {}_1^2\mathrm{H} + e^+ + \nu + 1.442 \text{ MeV} \\
{}_1^1\mathrm{H} + {}_1^2\mathrm{H} &\rightarrow {}_2^3\mathrm{He} + \gamma + 5.493 \text{ MeV}
\end{aligned}
$$

After both of these processes occurs twice, the cycle can take one of four different "branches," the most common of which, in the sun, is

$$
{}_2^3\mathrm{He} + {}_2^3\mathrm{He} \rightarrow {}_2^4\mathrm{He} + 2\,{}_1^1\mathrm{H} + 12.859 \text{ MeV}
$$

The entire cycle (referred to as the proton-proton cyle) can be summed up as

$$
4\,{}_1^1\mathrm{H} \rightarrow {}_2^4\mathrm{He} + 2e^+ + 2\nu
$$

The two positrons annihilate with two electrons, converting altogether into four photons which carry away 26.14 MeV, while the two neutrinos carry away 0.59 MeV. The total energy released is 26.73 MeV.

4. Calculate the energy released in the fusion process

$$
{}_2^4\mathbf{He} + {}_2^4\mathbf{He} + {}_2^4\mathbf{He} \rightarrow {}_6^{12}\mathbf{C}
$$

5. Stars hotter than the sun follow the carbon-nitrogen-oxygen (CNO) or Bethe cycle:

$$
\begin{aligned}
{}_1^1\mathbf{H} + {}_6^{12}\mathbf{C} &\rightarrow {}_7^{13}\mathbf{N} \\
{}_7^{13}\mathbf{N} &\rightarrow {}_6^{13}\mathbf{C} + e^+ + \nu_e \\
{}_1^1\mathbf{H} + {}_6^{13}\mathbf{C} &\rightarrow {}_7^{14}\mathbf{N} \\
{}_1^1\mathbf{H} + {}_7^{14}\mathbf{N} &\rightarrow {}_8^{15}\mathbf{O} \\
{}_8^{15}\mathbf{O} &\rightarrow {}_7^{15}\mathbf{N} + e^+ + \nu_e \\
{}_1^1\mathbf{H} + {}_7^{15}\mathbf{N} &\rightarrow {}_6^{12}\mathbf{C} + {}_2^4\mathbf{He}
\end{aligned}
$$

Calculate the energy released in this cycle. [Hint: Add all the reactions together, cancel common terms on opposite sides of the arrow, and come up with a summary or equivalent equation. Then compute the Q-value.]

Notice that the carbon is regenerated at the end of the cycle; it acts as a kind of catalyst.

6. Estimate (Calculate) the energy released per kilogram of hydrogen atoms consumed during the CNO cycle.

7. **The sun releases around 4×10^{26} W. If it did so by the CNO cycle, at what rate would hydrogen be consumed (in terms of kilograms of hydrogen per second)?**

8. **Show that the proton-proton and CNO cycles are equivalent (compare summary equations).**