Week 14: Particle Physics II (Undergraduate)

[Adapted from Chapter 2 of Griffiths, *Introduction to Elementary Particles*, 2010]

As best we know at the moment, the fundamental constituents of nature undergo only four kinds of interactions: strong, weak, electromagnetic, and gravitational. As evident in the table, the gravitational interaction is so much weaker than the other three that the Standard Model ignores it in describing the behavior of nature's fundamental constituents.

Interaction	Coupling Constant	Relative Strength	Typical Lifetime	Range	Sign	Intermediate Boson(s)
Strong	13	1	10^{-23} s	$< 10^{-15}$ m	$\pm$	gluons
Electromagnetic	1/137	$\sim 10^{-2}$	10^{-16} s	∞	$\pm$	photon
Weak	3×10^{-12}	10^{-13}	10^{-8} s	$< 10^{-18}$ m	$\pm$	$W^{\pm}$, Z^0
Gravitational	10^{-40}	10^{-38}	–	∞	–	graviton (?)

Strong interactions occur only between quarks (and therefore hadrons) and between the gluons which mediate the interaction. Electromagnetic interactions occur between all charged objects, including fundamental particles. Weak interactions occur between all fermions (leptons and quarks), and (for what it's worth), everything interacts gravitationally. Particle physics does not deal with gravitation, despite its universality. Relative to the interactions at accessible energies (and distances), it is much too weak to make an impact. The remaining interactions are described in the Standard Model in the language of quantum field theory, a merger of quantum mechanics with special relativity. Each interaction is attributed to a potential field, which is quantized, manifesting as one or more mediating bosons.

The strong interaction is described by Quantum Chromodynamics (QCD), the electromagnetic interaction by Quantum Electrodynamics (QED), and the weak interaction by what might be called Quantum Flavor Dynamics (QFD). The Standard Model combines the last two into Electroweak theory and tacks on QCD.

1. **By inspection of lifetimes and typical modes of transformation (see week 13 exercises 1 - 3), infer which interactions are responsible for the transformations of the particles in the**
   (a) **spin-$\frac{1}{2}$ baryon octet;**
   (b) **spin-$\frac{3}{2}$ baryon decuplet;**
   (c) **pseudoscalar (spin-0) meson nonet.**

2. **The $\phi(1020)$ meson is a $s\bar{s}$ resonance, with a lifetime of about 10^{-23} s. It's predominant modes of transformation are $\phi(1020) \to K^+ + K^-$, $\phi(1020) \to K^0 + \overline{K^0}$, and $\phi(1020) \to \pi^+ + \pi^- + \pi^0$.**
   (a) **Surmise the interaction(s) responsible for these transformations.**

(b) **Recall that the Q-value for a reaction $P \to D_1 + D_2 + D_3 + \cdots$ is**

$$Q = K_1 + K_2 + K_3 + \cdots - K_P = M_P - (M_1 + M_2 + M_3 + \cdots)$$

where P indicates parent, D daughter, K kinetic energy, and M mass. Compute the Q-values of the $\phi(1020)$ meson's transformations given that $M_\phi = 1020$ MeV, $M_{K^\pm} = 494$ MeV, $M_{K^0} = 498$ MeV, $M_{\pi^\pm} = 140$ MeV, and $M_{\pi^0} = 135$ MeV.

Interaction strengths and unstable particle lifetimes are characterized by dimensionless coupling constants: at low energies, coupling constants are proportional to interaction strength and inversely proportional to lifetime.

The most important calculational tool in the field theories of particle physics is the Feynman diagram. The convention followed here is that **time progresses from left to right** in the diagrams, while **nothing in the diagrams is indicative of positions in space**.[1] Note also that **antiparticles are indicated by arrows going backward in time.** Although their symbols in the diagrams are those of particles, their "anti-ness" is indicated by their arrows going backward in time.

A Feynman diagram represents a series of rules for calculating a physical quantity. To analyze a process, all diagrams that can contribute to that process should be identified (drawn). This amounts to creating every possible way that the inbound external lines can result in the outbound external lines, regardless of the number of internal lines and vertices. External lines in the diagrams represent observable particles. Internal lines in the diagrams represent "virtual particles," which cannot be observed. External lines exhibit the physically observable process; internal lines describe the underlying mechanism.

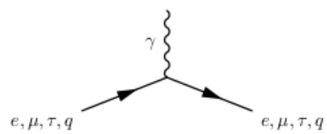
Each diagram is evaluated by following Feynman rules, which, among many other things, enforce momentum and energy conservation at each vertex–and therefore for the entire diagram. The evaluations of each relevant diagram are summed, and the sum represents the amplitude for the physical process represented by the external lines. The probability that the process occurs (that is, the cross-section) is the square of the amplitude.

The number of such diagrams is infinite. What may–or may not–save the day, depending on the interaction, is that each vertex in a diagram contributes one multiplicative factor of the interaction's coupling constant, which, again relates to the strength of the interaction. If the factor is significantly less than 1, the contribution from additional vertices decreases multiplicatively with number, so the number of diagrams that need to be analyzed depends on the precision required. If the coupling constant is equal to or greater than one, diagrams with additional vertices contribute at least as much to the amplitude as do those with fewer vertices.

[1] The more common convention for Feynman diagrams posits time progressing from the bottom of the diagram to the top, and spatial evolution represented from left to right, but I find this to be inconsistent and confusing in practice.

The coupling constant in QED is $\alpha = 1/137$, so additional vertices from ever more complicated diagrams contribute less, by the factor α for each vertex, to the amplitude than simpler diagrams. The contribution to the probability is roughly this factor squared.[2]

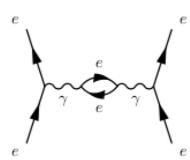
All electromagnetic phenomena are reducible to a charged particle emitting or absorbing a photon:

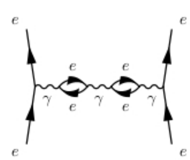


More complicated processes are diagrammed by incorporating additional primitive vertices. For example, Coulomb repulsion (known as Møller scattering) is represented by two diagrams, both of which must be included in the calculation of the cross-section:

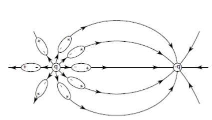
[2]It should be noted that α is not a constant at short distances (or as seen by probes of higher energy), but increases the closer the source charge. This occurs because the vacuum, whose lowest energy state is not zero, behaves like a dielectric, producing virtual e^+e^- pairs, which shield a "bare" charge by polarizing the region around it.

The effect is calculated with so-called loop diagrams,

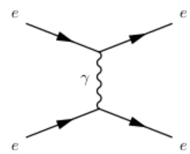


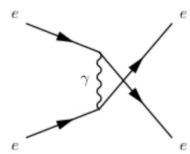


in which each loop is an e^+e^- pair, with the appropriate side of the loop drawn to the bare charge. This so-called vacuum polarization partially screens the charge, reducing the field.

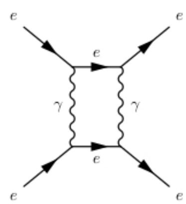


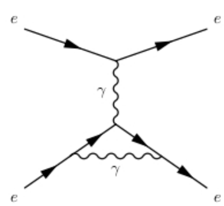
At distances less than around the electron Compton wavelength, $\lambda_C = hc/m_e = 2.43 \times 10^{-12}$ m, the effective charge increases. Obviously, this isn't what we measure as the unit charge. The effect is seen with high-energy scattering, and is only exhibited on the macroscopic scale by the Lamb shift in atomic spectra.

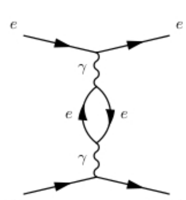


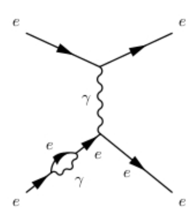


In fact, there are additional diagrams, requiring additional primitive vertices, and therefore additional factions of 1/137. For example:

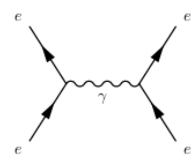


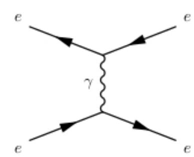






Rotating and/or twisting one Feynman diagram frequently results in the diagram of a different process. Such a topoligical reconfiguring of the Møller scattering diagrams yields the diagrams for the Coulomb attraction between, say, an electron and a positron (Bhabha scattering):





Again, both diagrams must be included in the analysis. Recall, that a particle line pointing backward in time represents the corresponding antiparticle.

Møller and Bhabha scattering are related by crossing symmetry. The occurrence of reaction

$$A + B \to C + D \tag{1}$$

implies that reactions expressed by moving one constituent of a viable reaction to the other side of process are dynamically possible and will occur if they are kinematically possible. That is, assuming no conservation law is violated, the following reactions will also occur:[3]

$$\begin{aligned} A &\to \bar{B} + C + D \\ A + \bar{C} &\to \bar{B} + D \\ \bar{C} + \bar{D} &\to \bar{A} + \bar{B} \end{aligned}$$

Notice that moving a particle to the other side of an interaction arrow transforms it into an antiparticle.

Calculations of diagrams related by crossing symmetry are nearly identical. From this point of view, the processes they represent are essentially distinct physical manifestations of the same process. Compton scattering, pair-production, and pair-annihilation are, despite obvious empirical differences, roughly indistinquishable theoretically.[4]

Pair annihilation $e^- + e^+ \to \gamma + \gamma$

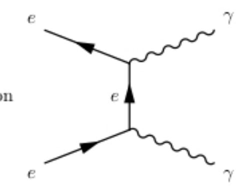
Pair production $\gamma + \gamma \to e^- + e^+$

[3] The reverse reaction, $C + D \to A + B$, is the result of the detailed balance principle–at equilibrium, an elementary process is in equilibrium with its reverse process–rather than crossing symmetry: $A + B \leftrightarrow C + D$.

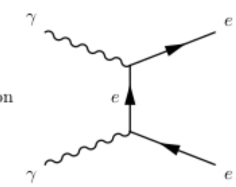
[4] The photon is its own antiparticle.

Compton scattering $\gamma + e^- \rightarrow \gamma + e^-$

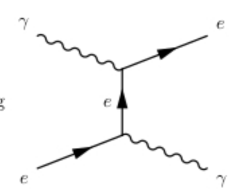
pair annihilation



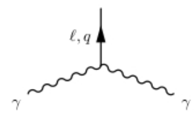
pair production

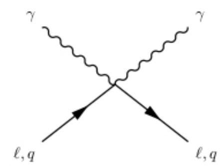


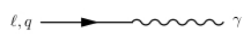
Compton scattering

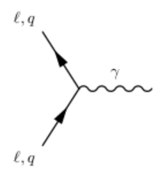


Some diagrams are impossible under electrodynamics:



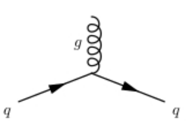




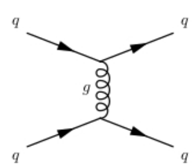


3. Explain why no processes in nature are described by the previous diagrams.

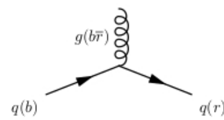
The primitive vertex of QCD is:



The basic strong interaction between (binding) two quarks is mediated by exchanging gluons



Electric charge is a single number (and a sign). Strong charge, or color, requires three symbols. At a quark-quark-gluon vertex, the flavor doesn't change, but the color may. However, color is always conserved, so the gluon must mediate the change.



Gluons carry one color and one anti-color. Given three colors, one might expect 9 combinations, but there are only 8.[5]

Photons don't carry electric charge, but gluons carry color, so, while photons do not couple to one another, gluons do, three or four at a time.[6] Another reason QCD is more difficult than QED is that the strong coupling constant, $\alpha_S > 1$, so additional vertices increase a diagram's contribution to the amplitude. On the other hand, α_S, like α, varies with energy, again due to vacuum (color) polarization.

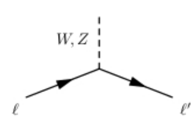
The energy scaling of α_S results from competition between virtual quark loops and virtual gluon loops (photon loops do not exist, because photons do not interact with other photons), that is, between quark polarization–which increases α_S–and gluon polarization–which decreases α_S. The outcome of the competition depends on the number f of quark flavors versus the number n of colors:

$$a = 2f - 11n$$

which equals -21 for 6 flavors and 3. Thus, $a < 0$, and so α_S decreases at short distances, a phenomenon known as asymptotic freedom.

Quarks and gluons are never found as free particles in nature. They are confined in colorless configurations of mesons (quark-antiquark pairs) and baryons (quark triplets). Quark confinement and the empirical manifestations of only colorless combinations is more difficult to explain than asymptotic freedom. These are long-range effects, where $\alpha_s > 1$, so perturbation theory and the Feynman method fail. Qualitatively, one assumes that in separating, the quark-gluon potential increases until it exceeds the energy threshold for the formation of additional quarks and gluons, and the state chooses the lower energy path: the potential snaps, as it were, and colorless objects form.

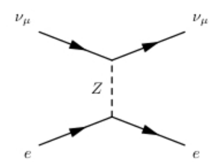
Other than gravity, which the Standard Model ignores, the weak interaction is the Standard Model's only universal interaction. In particular, neutrinos, being neutral leptons, interact only through the weak interaction. Reminiscent of the Yukawa interaction, the weak interaction proceeds through both charged and neutral modes (charged and neutral currents). Each diagram is a combination of weak interaction primitives.

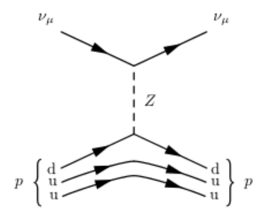


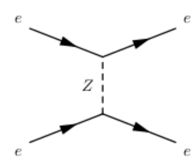
[5]If you know some group theory, the group $SU(3)$ has $n^2 - 1 = 8$ generators.

[6]Again, if you know group theory, the symmetry group of QED, $U(1)$, is abelian, while that of QCD, $SU(3)$, is not.

Neutral weak interactions include:



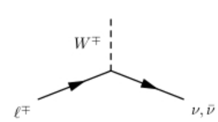




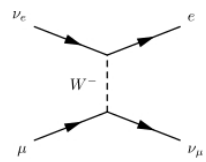
Note that any process mediated by a photon can also be mediated by a Z^0. QFD diagrams must be added to QED diagrams in the theoretical description of the process, and, consequently, the predicted transformation rate and cross-section are altered in a small, but measureable way from the predictions of QED alone. Precision experiments have measured the effect in, for example, $e^- + e^+ \to \mu^- + \mu^+$ Bhabha scattering and parity (mirror symmetry) violation in atomic, electromagnetic processes. Neutral currents, though, are most evident in interactions involving neutrino scattering.

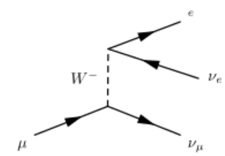
Charged weak interactions are the only known interactions that change the flavor of interaction particles. They alone produce transformations in which quantum numbers change, unlike the rearrangement of quarks in strong interactions and electromagnetic pair production and pair annihilation.

The weak primitive involving leptons looks like:



In the conversion of a charged lepton into a neutrino or vice versa due to the absorption or emission of a $W^\pm$ intermediate boson, the family is not changed; the evidence so far indicates that, except in neutrino oscillations, lepton family is a conserved quantity.





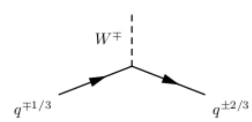
The neutrino-muon scattering of the first diagram is not likely, and such an experiment would be prohibitively difficult to perform, but the second diagram–a topological twist of the first diagram–yields the cleanest of all charged weak transitions, muon (beta) decay.

4. **The μ^+ lepton has a lifetime of around 2×10^{-6} s.**

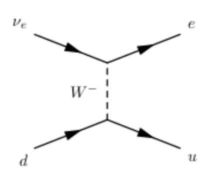
   (a) **Draw the Feynman diagram for this transition.**

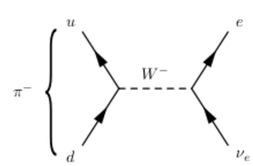
   (b) **What are the final state particles of this transition?**

The weak primitive involving quarks looks like:



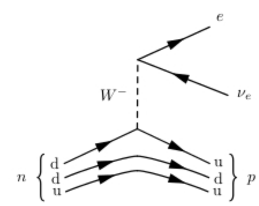
A representative, well-studied, charged weak tranformation involving quarks is the i“semileptonic” process in which, for example, a d-quark converts to a u-quark with the other vertex being a leptonic transition.



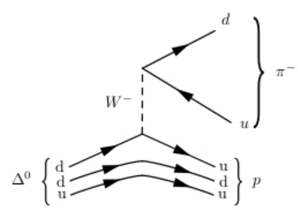


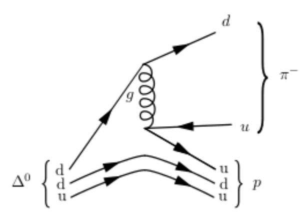
Because quarks are always confined, the bare quark reaction suggested by the first diagram will never occur, but the charged pion (lifetime around 10^{-8} s) is a common particle. It's transformation into an electron and an electron antineutrino is shown in the second diagram. Note, however, that the muon current is even more common than the electron current shown, due to a phenomenon referred to as helicity conservation.

A topological twist of the first diagram and some additional spectator quarks (quarks that remain unchanged by the interaction) yield neutron (beta) decay:



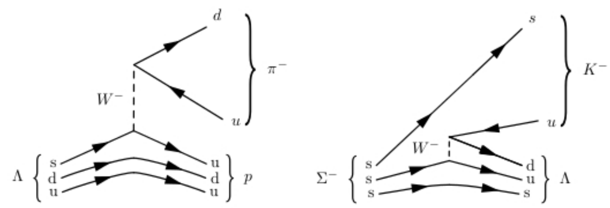
Charged weak interactions need not always involve leptons; weak hadronic transformations (involving only quarks) also occur.





But the strong interaction (lower diagram) is far more likely to bring about this transformation.

As it does with lepton flavor, charged weak interactions change quark flavor, but, unlike in the case of leptons, that change, though less likely the further away, may cross family lines. A charm quark is most likely to transition to a strange quark, but may transition to a down quark.



How is it that lepton transitions conserve family number, while quark transistions do not? The Standard Model answers this question by postulating that the charged weak interaction couples not to the quarks which couple to the strong interaction and have mass through the Higgs), but to linear combinations of these, mixed by the Cabibbo-Kobayashi-Maskawa (CKM) matrix:

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix} \tag{2}$$

That is, as far as the weak interaction is concerned, the families are

$$\begin{pmatrix} u \\ d' \end{pmatrix} \quad \begin{pmatrix} c \\ s' \end{pmatrix} \quad \begin{pmatrix} t \\ b' \end{pmatrix} \tag{3}$$

rather than

$$\begin{pmatrix} u \\ d \end{pmatrix} \quad \begin{pmatrix} c \\ s \end{pmatrix} \quad \begin{pmatrix} t \\ b \end{pmatrix} \tag{4}$$

and, with these linear combinations, 'weak' families are conserved.

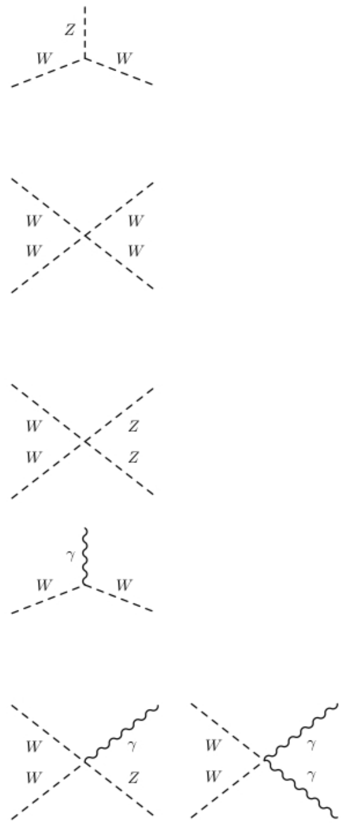
The square of a matrix element gives the relative probability of the particular transition occuring. If the CKM matrix were a unit matrix, then $d' = d$, $s' = s$, and $b' = b$, and there'd be no transitions between (strong interaction) families. Instead, nature has it that the matrix is unitary with (small) off-diagonal elements:

$$\begin{pmatrix} 0.97446 & 0.22452 & 0.00365 \\ 0.22438 & 0.97359 & 0.04214 \\ 0.00896 & 0.04133 & 0.999105 \end{pmatrix} \tag{5}$$

where uncertanties are in the last two digits.

For example, $d' = V_{ud}d + V_{us}s + V_{ub}b = 0.97446d + 0.22452s + 0.00365b$, so, of all weak charged transitions involving the d' quark (in the up-down$'$ quark family), nearly 95% will involve the down quark, while 5% will be with the strange quark, and a little over 1×10^{-3}% will be with the bottom quark.

It was noted that gluons couple to themselves as well as with quarks. Similarly, the weak mediating bosons couple to themselves, while $W^{\pm}$, being charged, also couple to the photon.



These result, as with those of the strong interactions, from the non-abelian nature of the weak interaction. In this case, however, such self-couplings are of little consequence empirically, because such contributions are surpressed by the small magnitude of the weak and electromagnetic coupling constants.

5. **The cascade-minus Ξ^- is a baryon comprised of two strange s quarks and one down d quark. It can, in principal, transform into either of these hadronic final states:**

   - $\Xi^- \to \Lambda^0 + \pi^-$
   - $\Xi^- \to n + \pi^-$

   The quark content of the Λ^0 baryon is uds; that of the π^- meson is $d\bar{u}$; and that of the neutron n is udd. Draw the Feynman diagrams of each of decay mode, and decide which is the more likely to occur. Explain.

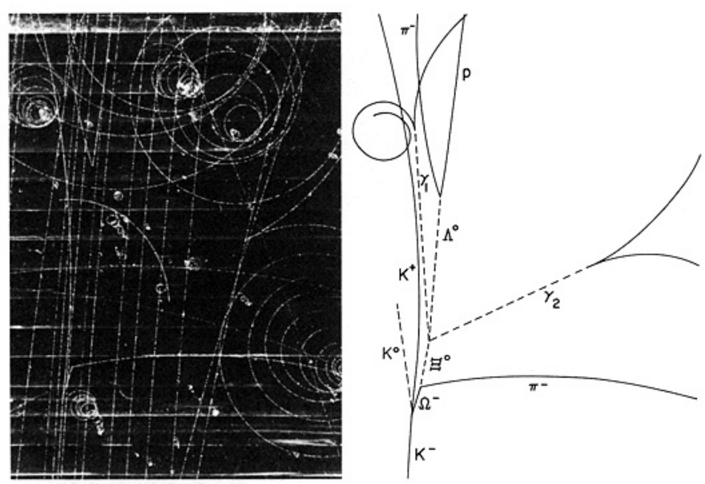
6. **Draw Feynman diagrams for the following D^0 ($c\bar{u}$) transitions:**

$$D^0 \to K^- + \pi^+$$
$$D^0 \to \pi^- + \pi^+$$
$$D^0 \to K^+ + \pi^-$$

Which is the most likely, which is the least? Explain.

[The quark content of the K^- meson is $s\bar{u}$; of the π^+ meson, $u\bar{d}$; of the π^- meson, $d\bar{u}$; and of the K^+, $u\bar{s}$.]

7. The image on the left below is a bubble chamber exposure of the first observed Ω^- baryon event (1964) at Brookhaven National Laboratory. On the right are traced out the tracks of the production and subsequent transformations. A K^- enters the bubble chamber from below.



(a) The initial reaction was $K^- + X \to K^0 + K^+ + \Omega^-$. What was X? What type of interaction was involved?

(b) Write the equation of each subsequent reaction and identify the type of interaction.

[The quark composition of the particles is

- K^-: $s\bar{u}$
- Ω^-: sss
- K^0: $d\bar{s}$
- K^+: $u\bar{s}$
- π^-: $d\bar{u}$

- **Ξ: *ssu***
- **Λ^0: *dus***
- ***p*: *uud*]**

8. **More than 99% of the time, the Ω^- transforms hadronically via the weak interaction. Identify and explain which of the following transformations is impossible?**

   (a) $\Omega^- \to \Xi^- + \pi^0$

   (b) $\Omega^- \to \Lambda^0 + K^-$

   (c) $\Omega^- \to \Sigma^- + K^0$

The most universal pattern of particle behavior is that every one of them transforms into some number of other particles unless one or more conservation laws prevents them. Preventing such transitions could be that no less massive particles exist, leaving, in the neutral case photons stable, and, in the charged case, electrons stable. Baryon number conservation, assuming it holds, leaves protons, as the lightest baryon, stable, while lepton number conservation leaves the lightest neutrino stable. Similarly for positrons, anti-protons, and the lightest anti-neutrino.

Aside from a handful of stable particles, then, everything else in the universe has a finite lifetime. Some are characterized by mean lifetimes, because they may transform in different ways. Each way has its own likelihood of occurring (of course, the sum of all likelihoods is 100%), called, in the Standard Model, a branching fraction. Calculating lifetimes and branching fractions are among the primary objectives of particle physics theory.

Lifetimes are more or less characteristic of the interaction leading to the transformation. Particles with the shortest lifetimes, around 10^{-23} s, tend to transform via the strong interaction, while particles with the longest lifetimes, ranging from around 10^{-13} s to 15 min, tend to transform via the weak interaction. Particles transformed by the electromagnetic interaction typically have lifetimes between those transformed by the strong or weak interactions, roughly 10^{-16} s.

As far as particle physics experimentalists are concerned, if the lifetime is long enough that it can be measured by track length or vertex displacement, the particle is "stable," so "only" strongly transforming states are unstable. Their lifetimes are measured indirectly by reconstructing their masses, measuring the width of the resulting mass distribution, and then estimating the lifetimes from the energy-time uncertainty principle.

$$\tau = \Delta t \geq \frac{\hbar}{2\Delta m}$$

As can be seen, the wider the mass distribution, the shorter the lifetime.

All interactions obey energy, (linear and angular) momentum, and charge conservation. Unlike energy and momentum conservation, charge conservation involves quantized values, and so a quantum number, Q = charge/e, where $e = 0.3$ in natural units, can be assigned to every particle. Q is conserved in all reactions and transformations. Note that if a particle has charge Q, its antiparticle has charge $-Q$.

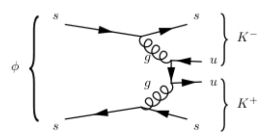
9. **Explain charge conservation in the threshold production of antiprotons discussed in the previous exercise set:**

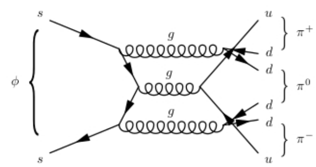
$$\boldsymbol{p + p \rightarrow p + p + p + \bar{p}}$$

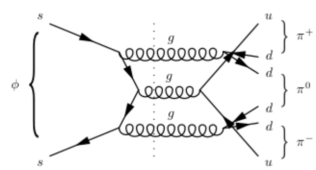
Other universally or partially conserved quantum numbers include baryon number, lepton number, isospin, strangeness, and parity.

Here is a list of the most important (for particle physics) conserved quantities. Each is conserved at every primitive vertex, and therefore by all reactions.

- Energy (all)
- Momentum (all)
- Angular momentum (all)
- Charge (all)
- Color (all) strong–zero in, zero out; others do not affect color
- Baryon number (all) Each quark has baryon number 1/3 (and each antiquark has baryon number -1/3).The total number of quarks + antiquarks is the same before and after each interaction. Baryons (three quarks) have baryon number 1; anti-baryons (three anti-quarks) have baryon number -1; mesons (quark, anti-quark pairs) have baryon number 0, as do all leptons.
- Lepton number (all) The strong interaction does not affect lepton number; the electromagnetic interaction has same in, same out; the weak interaction has lepton in, lepton out, essentially always in the same family–lepton family number is approximately conserved, but neutrinos oscillate, so lepton families may involve linear combinations, just like quarks.
- Flavor Strong, electromagnetic, and neutral weak interactions only; charged weak interactions change flavor.
- OZI [Okubo, Zweig, Iizuka] (relevant only to strong interactions) If initial state particles connect to final-state particles only by gluon lines in a Feynman diagram, the process is suppressed.







10. **The Feynman diagrams show two of the three $\phi(1020)$ transformations whose Q-values were calculated in the second exercise. Normally, Q-value and transformation likelihood are directly related. Look up the branching fractions for these $\phi(1020)$ decay modes in**

    `https://pdg.lbl.gov/2022/web/viewer.html?file=../listings/rpp2022-list-phi-1020.pdf`

    to see if this is the case. If it isn't, explain.

11. **(a) Which conservation laws would the reaction $p \rightarrow \pi^0 + e^+ + e^-$ violate?**

    (b) Which conservation laws would the reaction $p \rightarrow \pi^0 + e^+$ violate?

12. **Which reaction in each pair is possible? Explain your choice.**

    (a) (strong interaction)

    - **i.** $p + p \rightarrow p + n + K^+$
    - **ii.** $p + p \rightarrow \Lambda^0 + K^0 + p + \pi^+$

    (b) (weak interaction)

    - **i.** $\Sigma^- \rightarrow \pi^- + n$
    - **ii.** $\Sigma^- \rightarrow \pi^- + p$

    (c) (strong interaction)

    - **i.** $\pi^- + p \rightarrow \Sigma^0 + \eta$
    - **ii.** $\pi^- + p \rightarrow \Sigma^0 + K^0$

(d) **(weak interaction)**

i. $\boldsymbol{n \rightarrow p + e^- + \nu_e}$

ii. $\boldsymbol{n \rightarrow p + e^- + \bar{\nu}_e}$

(e) **(strong interaction)**

i. $\boldsymbol{p + p \rightarrow K^+ + \Sigma^+}$

ii. $\boldsymbol{p + p \rightarrow K^+ + p + \Lambda^0}$