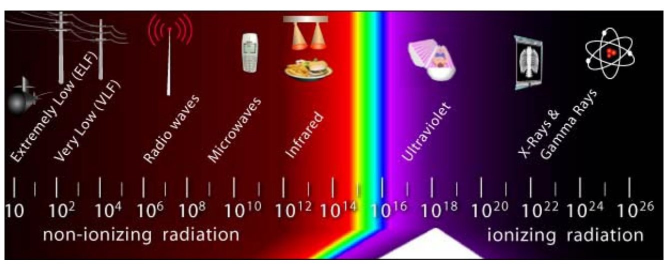
Week 3: Interactions Between Electromagnetic Radiation and Matter (Undergraduate)

Any particle, massive or not, emitted in atomic and sub-atomic processes is referred to as radiation. Radiation of sufficiently high energy can ionize (cause electrons to be ejected from) atoms and molecules. Some can disrupt nuclei. Ionizing radiation includes electromagnetic radiation, such as photons and gamma-rays, and particulate radiation, such as alpha- and beta-particles, neutrons, and protons.

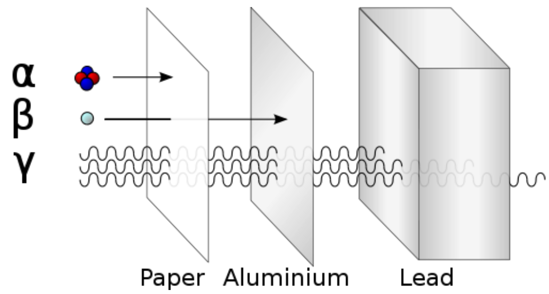
Ionization occurs when the interaction between the radiation and an atom or molecule of matter transfer sufficient energy to the atom or molecule to eject an electron. This is known as primary ionization. If the ejected electron has enough energy to eject a second electron, either from its parent or another atom or molecule, the process is called secondary ionization.

Interactions between radiation and matter need not result in ionization. It may happen that the radiation scatters elastically, transferring no energy and leaving the atom or molecule in its ground state. It may also happen that only enough energy is tranferred to excite the atom or molecule, which then emits (low energy) photons as it returns to its ground state.

Only radiation of sufficient energy can ionize matter. For example, the wavelengths of radio waves and microwaves are too long (frequency too low) to ionize matter. Even thermal neutrons, which can induce fission and participate in fusion processes, cannot ionize atoms or molecules.



Ionizing power is the term for the number of ionized pairs produced by radiation per centimeter of passage through matter. Alpha-particles have the most ionizing power, and gamma rays have the least. Ionizing power and penetrability are roughly inversely related: the greater the ionizing power, the less distance through matter. Thus, a sheet of paper or a layer of clothing is sufficient to stop alpha-particles. Beta-particles will penetrate several millimeters into solids and liquids and tens of centimeters of gas. Gamma rays penetrate most materials, losing significant energy or being absorbed only in heavy, dense metals, such as lead.



Electromagnetic radiation interactions with matter may take any of a number of forms, depending on the energy (wavelength, frequency) of the radiation. Let $\hbar\omega_0$ signify the energy required to ionize the target atom.

Energy	Rayleigh Scattering and Photoelectric Effect	Thomson Scattering	Compton Scattering	Pair Production
Range	$\hbar\omega \leq \hbar\omega_0$	$\hbar\omega_0 \ll \hbar\omega \ll m_e c^2$	$\hbar\omega \approx m_e c^2$	$\hbar\omega > 2m_e c^2$
Scale	eV	keV	MeV	$\geq$ MeV
Spectrum	visible - ultraviolet	X-rays	gamma-rays	high-energy gamma-rays

While the other processes require a full quantum-mechanical treatment, the magnitude of Rayleigh and Thomson scattering cross-sections can be approximated by modeling an atomic electron as oscillating around its "rest" position with a natural frequency $\omega_0 = \sqrt{\frac{k}{m_e}}$, where m_e is the electron mass, and k is the "spring constant". Electromagnetic radiation of frequency ω (energy $\hbar\omega$) impinging on the electron acts as a driving force $F = eE(t)$, where $E(t) = E_0 \sin(\omega t)$ is the electric field. If x_e is the electron position relative to its rest position,

$$m_e \ddot{x}_e = -kx_e - eE(t)$$
$$\Rightarrow \ddot{x}_e + \omega_0^2 x_e = -\frac{e}{m_e}E(t)$$

A solution of the form $x_e(t) = A\sin(\omega t)$ leads to:

$$(\omega_0^2 - \omega^2)A = -\frac{e}{m_e}E_0$$
$$\Rightarrow A = \frac{1}{\omega^2 - \omega_0^2}\frac{e}{m_e}E_0$$

The acceleration is $\ddot{x}_e = -\omega^2 A\sin(\omega t)$, and, therefore, the mean square acceleration is

$$< \ddot{x}_e^2 >= \frac{1}{2}\left(\frac{\omega^2}{\omega_0^2-\omega^2}\frac{e}{m_e}E_0\right)^2$$

Accelerated charges radiate, with power proportional to the acceleration squared, according to the (non-relativistic) Larmor formula (in SI units):

$$P = \frac{e^2}{6\pi\varepsilon_0 c^3}a^2$$
$$\Rightarrow < P > = \frac{1}{12\pi\varepsilon_0}\left(\frac{e^2}{m_e c^2}\right)^2 \frac{\omega^4}{(\omega_0^2-\omega^2)^2}cE_0^2$$

Radiation intensity, which equals the Poynting vector, can be written in terms of the amplitude of the electric field as

$$I = \frac{1}{2}\varepsilon c E_0^2$$

($c = E/B = 1/\sqrt{\mu_0\varepsilon_0}$).

Recall that intensity = power / area, so $P = \sigma I$, as the cross-section is the relevant area for interactions between electromagnetic radiation and matter. Therefore,

$$\sigma = \frac{8\pi}{3}\left(\frac{e^2}{4\pi\varepsilon_0 m_e c^2}\right)^2\left(\frac{\omega^2}{\omega_0^2-\omega^2}\right)^2 = \pi r_e^2\frac{8}{3}\left(\frac{\omega^2}{\omega_0^2-\omega^2}\right)^2$$

where $r_e \equiv \frac{1}{4\pi\varepsilon_0}\frac{e^2}{m_e c^2} \approx 2.8$ fm is known as the "classical electron radius," "Lorentz radius," or "Thomson scattering length," a length scale consistent with the energy necessary to assemble 1.6×10^{-19} Coulombs into a sphere. It's a useful length because it characterizes atomic-scale interactions with electrons. Notice that πr_e^2 would be the classical, mechanical cross-section for such a sphere. It is modified in the case of atomic-scale electromagnetic interactions by a frequency (or wavelength) dependence.

Very low-energy electromagnetic radiation, $\omega \ll \omega_0$, will not ionize an atom or molecule. In fact, the target is left in the same energy state it was in before the interaction (elastic scattering). In this limit, the frequency factor is $\frac{\omega^2}{\omega_0^2-\omega^2} \approx \frac{\omega^2}{\omega_0^2}$, so the cross-section becomes

$$\sigma_R = \frac{8\pi}{3}\left(\frac{e^2}{4\pi\varepsilon_0 m_e c^2}\right)^2\frac{\omega^4}{\omega_0^4}$$

This is known as Rayleigh scattering. It has a very strong frequency (wavelength) dependence, as exhibited by the coloring of the sky at various times of the day.

1. **Electromagnetic waves with wavelengths between 450 nm and 485 nm are perceived as blue light. Wavelengths between 625 nm and 700 nm are perceived as red light. How much more likely is blue light than red light to be scattered by air molecules?**

2. (a) **Why is the sky blue on a sunny day?**
   (b) **Why is sunset reddish-orange?**
   (c) **Why are clouds white or gray?**
   (d) **Why are stop lights and other warning lights red?**

When $\hbar\omega \lesssim \hbar\omega_0$, the dominant process is the photoelectric effect. This process involves individual photons, and a classical approach no longer works–quantum mechanics is necessary. In this view, a single target electron absorbs an individual photon. If the energy thus absorbed is greater than the energy binding the electron to the atom or molecule, then the electron can free itself from the atom or molecule. The magnitudes of binding energy depend on the target material and the configuration of each electron in the material. The difference between the energy absorbed by the electron and the energy binding the electron to its parent atom manifests as the electron's kinetic energy, once emitted. The smallest energy binding electrons (the least tightly bound electrons) is referred to as the work function, ϕ.

$$K_{\text{max}} = \hbar\omega - \phi$$

This implies that there is a threshold energy, $\hbar\omega_0$, below which no electrons will be emitted, regardless of the intensity of the electromagnetic radiation.

The full quantum mechanical treatment shows a very strong Z dependence:

$$\sigma_{pe} \propto \frac{Z^n}{(\hbar\omega)^3}$$

where $4 < n < 5$. The photoelectric effect therefore is more likely to occur with high Z targets, but becomes less likely quickly as $\hbar\omega \gg \hbar\omega_0$.

3. (a) **A photoelectric effect experiment on a pure sample of some metal finds that the largest wavelength light that emits an electron is 562 nm. What is the metal's work function (in electron volts)?**
   (b) **What is the maximum electron kinetic energy when this sample is illuminated with ultraviolet light ($\boldsymbol{\lambda = 250}$ nm)?**
   (c) **Of what element might this sample be?**

When $\hbar\omega = \hbar\omega_0$, this model blows up. Physically, this indicates a resonance condition, and the system becomes disrupted. The target ionizes, and there is no more natural frequency.

Electromagnetic radiation with energies

$$\hbar\omega_0 \ll \hbar\omega \ll m_e c^2,$$

referred to in the spectrum as X-rays, is sufficient to ionize the target but not high enough to impart the electron with relativistic energy. If electromagnetic radiation in this energy range interacts elastically with a quasi-free electron (for example, one in the conduction band of a metal), it can accelerate the electron, which, in turn, emits electromagnetic radiation at the same frequency as the incoming radiation, leaving the target's energy unchanged. This sort of reaction is referred to as Thomson scattering, after J. J. Thomson.

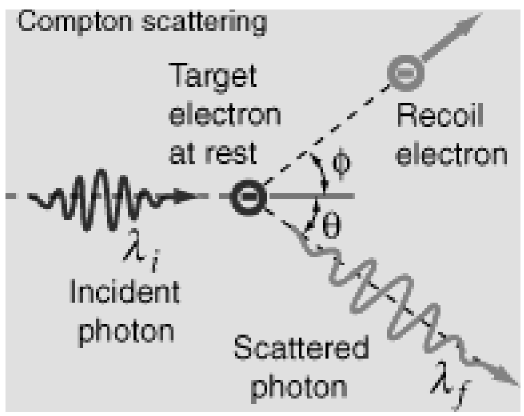
In the limit $\omega_0 \ll \omega$, so that $\frac{\omega^2}{\omega_0^2 - \omega^2} \approx -1$, the scattering cross-section becomes

$$\sigma_T = \frac{8\pi}{3}\left(\frac{e^2}{4\pi\varepsilon_0 m_e c^2}\right)^2 = \frac{8}{3}\pi r_e^2$$

Notice that the Thomson scattering cross-section has no frequency dependence, but, instead, is a constant in this energy range.

4. What is the value of σ_T in meters, barns, natural units?

High-energy X-rays and low-energy gamma-rays, $\hbar\omega \approx m_e c^2 \approx 10^6$ eV, can ionize atoms and molecules. The interaction between such radiation and a quasi-free electron gives rise to Compton scattering.



5. Treating X-rays and gamma-rays as photons (particles), applying conservation of momentum and energy, and employing the Planck, de Broglie, and frequency-wavelength relationships ($E = h\nu = \hbar\omega$, $\vec{p} = h/\vec{\lambda} = \hbar\vec{k}$, $c = \nu\lambda = \omega/k$)[1], and the relativistic energy-momentum relation ($m^2 = E^2 - |\vec{p}|^2$), determine the energy of the outgoing photon, $\hbar\omega'$, as a function of the incoming photon energy, $\hbar\omega$ and photon scattering angle, θ, after Compton scattering.

Because energy has been transferred from the projectile to the target, and the target is disrupted, Compton scattering is inelastic, sometimes referred to as incoherent, since the incoming and outgoing waves can never be in a fixed phase relationship.

[1] $\boldsymbol{k \equiv 2\pi/\lambda}$ is the angular wave number

6. (a) **In a Compton scattering experiment, a 0.5-MeV photon interacts with an electron at rest. The electron is ejected with a kinetic energy of 0.1 MeV. What is the final energy of the photon?**

   (b) **What is the photon scattering angle, $\boldsymbol{\theta}$?**

Except for the energy of the electromagnetic radiation, Compton and Thomson scattering are very similar. At the higher energy, though, the radiation must be treated as a quantum, a particle, the photon, and calculating the cross-section, for example, requires a full quantum mechanical treatment (in fact, to get the cross-section right, quantum field theory is necessary). The outcome is, to first order,

$$\sigma_C = \sigma_T \frac{m_e c^2}{\hbar\omega}$$

Notice that the Compton cross-sections decreases at high energy.

In case the question hasn't arisen yet, it's worth asking how a photon can scatter from an electron? The usual metaphor of two billiard balls colliding isn't very helpful, since these are not mechanical interactions. Even the Coulomb interaction doesn't help, since the photon is neutral. A more modern manner of speaking is to say that the electron absorbs the photon (and therefore its energy), is accelerated (recoils), and immediately emits a photon, not necessarily of the same wavelength or in the same direction as the orginal photon, and again recoils. If the emitted photon has the same energy as the absorbed photon, the two recoils "cancel," the system is left in its original state, and the interaction, again, is referred to as elastic. If the emitted photon has lower energy (by conservation of energy, it can't have greater energy if the electron is originally at rest), then the electron's energy state has changed, and the interaction is referred to as inelastic. This is all described mathematically in Quantum Electrodynamics, so this way of speaking is, as is usually the case with quantum phenomena, figurative.

A high-energy gamma-ray can convert into a matter-antimatter pair (an electron and a positron, for example) in the vicinity of a nucleus, or some other quantum object. For the process, called pair production, to occur, another object must interact with the gamma-ray such that momentum and energy are conserved. The energy of the gamma-ray must at least equal the total mass of the pair. Any extra energy manifests as the pair's kinetic energy.

If the process occurs in the vicinity of a heavy nucleus, the nucleus can carry away an appreciable amount of the gamma-ray's initial momentum, but, because $K \approx p^2/2M$, where M, the mass of the nucleus, is typically much greater than the momentum, the kinetic energy is negligible. Because the mass of the nucleus would appear on both sides of the conservation of energy equation, this mass, too, can be ignored.

$$\hbar\omega = 2mc^2 + K_+ + K_-$$

where m is the mass of either daughter particle (matter-antimatter pairs have identical masses), and $K_+(_-)$ is the kinetic energy of the positive (negative) member of the pair.

7. **What is the longest wavelength of a photon that can produce an electron-positron pair? [$m_e = 0.511$ MeV]**

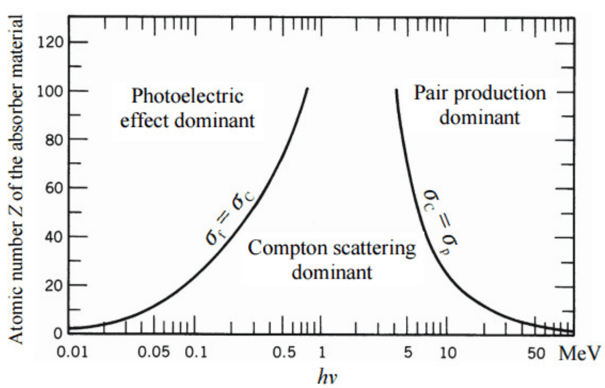
8. **An electron-positron pair is created from a 3×10^{-4} nm wavelength photon. If the positron's kinetic energy is twice the electron's, what are the kinetic energies of the two particles?**

Quantum Electrodynamics shows that pair production and Compton scattering are closely related. The pair production cross-section, like that of Compton scattering, is proportional to $1/\omega$.

These electromagnetic interactions–the photoelectric effect, Compton scattering, and pair production–combine to reduce the flux (intensity) of a photon beam as it passes through material. The reduction follows the usual exponential decay (here, attenuation) law:

$$I = I_0 e^{-\mu x}$$

where I_0 is the incident intensity, and μ is the linear absorption coefficient. μ is a constant characteristic of the material at a given energy photon. μ varies with energy for a given material because the cross-sections for the different interactions vary with ω as well as Z.



9. **What fraction of a photon beam remains after passing through material whose thickness equals two half-thicknesses?**

10. **Shielding of a photon beam aims to reduce the intensity by a factor of 5. Two different materials are available, one, A, which has an absorption coefficient $\mu_A = 0.044$ mm^{-1}, must be twice as thick as the other, B, which has an absorption coefficient $\mu_B = 0.056$ mm^{-1}. How thick should each material be?**