Week 4: Interactions Between Charged Particle Radiation and Matter (Undergraduate)

Much of the following text is adapted from Cappellaro, P. (2022, October 12). "Interaction of Radiation with Matter," at `https://phys.libretexts.org/Bookshelves/Nuclear_and_Particle_Physics/Book%3A_Introduction_to_Applied_Nuclear_Physics_(Cappellaro)/08%3A_Applications_of_Nuclear_Science_(PDF_-_1.4MB)/8.01%3A_Interaction_of_Radiation_with_Matter`

Recall that the likelihood of interaction is quantified by a cross-section. To recapitulate, the classical meaning of a cross-section is the area of impact presented to a projectile. A spherical target whose radius is r has a mechanical cross-section, in everyday dimensions, of $\sigma = \pi r^2$, as that is the area of the sphere's circular cross-section. Nuclei have radii $r = r_0 A^{1/3}$, where $r_0 = 1.2{-}1.4$ fm is an empirically determined constant and A is the mass number (number of nucleons). If nuclei where classical spheres, and interactions with them were only mechanical, then the cross-section for nuclear interactions would be $\sigma = \pi r_0^2 A^{2/3}$ fm^2. The interaction cross-section of $^{137}_{56}$Ba, for example, would be $\sigma \approx 100$ fm$^2 \equiv 1$ barn (1 b $= 100$ fm$^2 = 10^{-28}$ m^2).

Interactions with atomic electrons are much more likely than interactions with nuclei. While the radii of most nuclei are between 5 and 10 fm (10^{-15} m), the radius of an atom is of the order of angstroms (10^{-10} m), so the atomic interaction cross-section is at least $(10^4)^2 = 10^8$ times larger than the nuclear interaction cross-section. On the other hand, electrons are very light compared to the masses of nuclei and of most projectiles, so the effect on the projectiles of atomic collisions will be less than the effect of nuclei.

Nuclei and atoms are not classical objects, however, nor are interactions with them mechanical. Rather, interactions with atoms (i.e, atomic electrons) are coulombic or weak, while interactions with nuclei are coulombic, weak, or strong. Because it's charged, the proton scattering cross-section is therefore larger than the neutron scattering cross-section. Both will be much larger than the scattering cross-section of a neutrino, which interacts only weakly (the weak interaction).

Consider a reaction of the form $A(x,\ y)A'$ $(x + A \to A' + y)$, where A and A' are heavy, stationary nuclei, x is an incoming projectile, and y is an outgoing particle. The cross-section is measured by the rate of detection of y, R_y, relative to the flux of projectiles x, Φ_x and to the number of target nuclei per unit area, n:

$$\sigma = \frac{R_y}{\Phi_x n}$$

Experiments are rarely capable of measuring total cross-sections. The finite sensitivity of detectors limit the detection of outgoing particles to finite regions of space at polar angle θ and aximuthal angle φ, which, at a certain distance from the interaction is called the solid angle, $d\Omega$. The rate measurement then leads to what is referred to as the differential cross-section:

$$\frac{d\sigma}{d\Omega} = \frac{R(\theta,\ \varphi)}{4\pi\Phi_x n}$$

The total cross-section can then be calculated by integrating over all angles:

$$\sigma = \int_{4\pi} \frac{d\sigma}{d\Omega} d\Omega = \int_0^{\pi} \sin\theta d\theta \int_0^{2\pi} d\varphi \frac{d\sigma}{d\Omega}$$

Obviously, measurements at various solid angles are done to check this extrapolation. If $\frac{d\sigma}{d\Omega}$ is found to be constant, then $\sigma = 4\pi \frac{d\sigma}{d\Omega}$.

Cross-sections often depend on projectile energy. Such dependence can reveal structural details of the target or projectile. With a detector sensitive to energy and covering the full 4π solid angle, the dependence can be determined by measuring the cross-section as a function of projectile energy, E_b,

$$\frac{d\sigma}{dE_b}$$

or, if the detector can cover only finite regions at a time, the dependence can be determined by measuring the double differential cross-section

$$\frac{d\sigma^2}{d\Omega dE_b}$$

Consider what happens to an alpha-particle interacting with an atomic electron through the Coulomb interaction. Assuming a non-relativistic, elastic interaction in which mometum and kinetic energy are conserved (assume the electron is initially at rest),

$$m_\alpha v_\alpha = m_a v'_\alpha + m_e v_e$$
$$m_\alpha v_\alpha^2 = m_a v'^2_\alpha + m_e v_e^2$$

where the prime identifies the α-particle's velocity "after the interaction."

Then

$$v'_\alpha = \frac{m_\alpha - m_e}{m_\alpha + m_e} v_\alpha$$
$$v_e = \frac{2m_\alpha}{m_\alpha + m_e} v_\alpha$$

And since $m_\alpha \gg m_e$, $v'_\alpha \approx \frac{m_\alpha}{m_\alpha + m_e} v_\alpha \approx v_\alpha$ and $v_e \approx 2v_\alpha$.

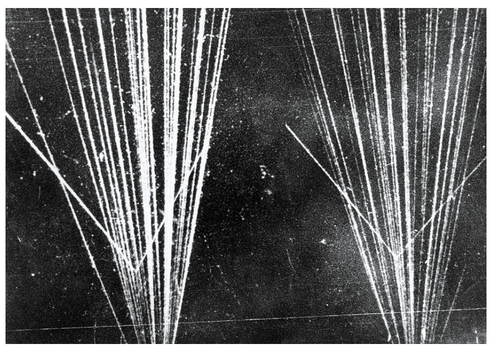
The alpha-particle loses kinetic energy to the electron:

$$\Delta K_\alpha = \frac{1}{2} m_e v_e^2 \approx \frac{1}{2} m_e (2v_\alpha)^2 = 2m_e v_\alpha^2 = 4\frac{m_e}{m_\alpha} K_\alpha$$
$$\Rightarrow \frac{\Delta K_\alpha}{K_\alpha} \approx \frac{m_e}{m_\alpha} \ll 1$$

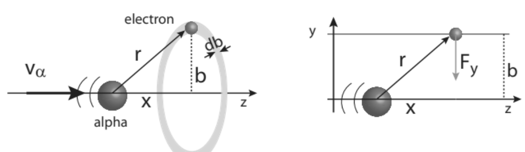
Thus a single interaction with an atomic electron hardly reduces an alpha-particle projectile's kinetic energy.

All of this implies that

1. Single-electron interactions barely perturb the trajectories of alpha-particle projectiles through matter.
2. To slow or stop an alpha-particle projectile through single electron collisions requires hundreds of such collisions.
3. Alpha-particles interact with many electrons concurrently, because the Coulomb interaction has infinite range.
4. The kinetic energy transferred to many of these electrons is sufficent to ionize atoms, thus making it possible to visualize the trajectories.



The energy lost per unit length is thus a more informative quantity than energy lost per collision.



Recall that impulse equals the change in momentum.

$$\Delta p = \int_0^t F dt$$

The alpha-particle-electron interaction is a non-relativistic electrostatic interaction. The figure shows the alpha-particle moving in the positive x-direction with an impact parameter b. Under the assumption the electron is at rest as the alpha-particle passes, forward and backward affects cancel, and the net (attractive) effect is in the transverse direction: $F_y = |F| \sin\theta$, if θ is the angle between the electron position and the alpha-particle's initial direction. Given that, in this geometry, $\sin\theta = b/r$, where $r = \sqrt{x^2 + b^2}$,

$$F_y = \frac{2e^2}{4\pi\varepsilon_0 r^2}\sin\theta = \frac{2e^2}{4\pi\varepsilon_0}\frac{b}{(x^2+b^2)^{3/2}}$$

Then the momentum change is (using $v_\alpha = dx/dt$ assumed to be constant; also assumed is that the electron is initially at rest),

$$\Delta p = p_e = \frac{2e^2}{4\pi\varepsilon_0 v_\alpha}\int_{-\infty}^{\infty}\frac{b}{(x^2+b^2)^{3/2}}dx = \frac{2e^2}{4\pi\varepsilon_0 b v_\alpha}\left.\frac{x}{\sqrt{x^2+b^2}}\right]_{-\infty}^{\infty} = \frac{4e^2}{4\pi\varepsilon_0 b v_\alpha}$$

An electron, whose transverse distance from the line of flight of the alpha-particle is b, therefore gains energy from the alpha-particle:

$$\Delta K_\alpha = \frac{p_e^2}{2m_e} = \frac{8e^4}{(4\pi\varepsilon_0)^2 m_e b^2 v_\alpha^2}$$

The total energy lost by an alpha-particle traversing a material depends on the number of such electron collisions. For each infinitesimal thickness dx of material,

$$-dE = \int_0^{N_e}\Delta K_\alpha dN_e = 2\pi dx\int_{b_{\min}}^{b_{\max}} n_e \Delta K_\alpha b\ db$$
$$\Rightarrow -\frac{dE}{dx} = \frac{16\pi e^4 n_e}{(4\pi\varepsilon_0)^2 m_e v_\alpha^2}\ln\left(\frac{b_{\max}}{b_{\min}}\right)$$

where $dN_e = n_e 2\pi\ b\ db\ dx$ is the number of electrons in an infinitesimal cylinderical shell of radius b, shell thickness db, and length dx; $n_e = N_A Z\rho/A$ is the number density of electrons; N_A is Avogadro's number, Z is the atomic number of the material, ρ is the density of the material, and A is the mass number of the material. A is also the mass in grams of one mole of the element ($A = M = N_A m$, where m is the mass of one atom). The integral is taken over all physical impact parameters.

It remains, in determining the alpha-particle energy lost per unit traversal length, to fix $b_{\max}$ and $b_{\min}$ in terms of measurable values. $b_{\max}$ is approximately the atomic radius, which itself can be approximated by the distance from the nucleus at which the Coulomb potential is equal to a quantity called the excitation potential $U_e \approx 10Z$ eV, where Z is the target's atomic number (this relationship is found empirically):

$$U_e = \frac{e^2}{4\pi\epsilon_0}\frac{1}{b_{\max}}$$
$$\Rightarrow b_{\max} = \frac{e^2}{10\ Z\ 4\pi\epsilon_0}$$

$b_{\min}$ we know is the minimum distance of approach in a head-on elastic collision. Since $m_\alpha \gg m_e$, $v_e \approx 2v_\alpha$, $K_e \approx 2m_e v_\alpha^2$ corresponding to the maximum Coulomb potential, $U_{\max} \approx \frac{2e^2}{4\pi\varepsilon_0}\frac{1}{b_{\min}}$. Therefore,

$$b_{\min} = \frac{2e^2}{4\pi\varepsilon_0}\frac{1}{2m_e v_\alpha^2}$$

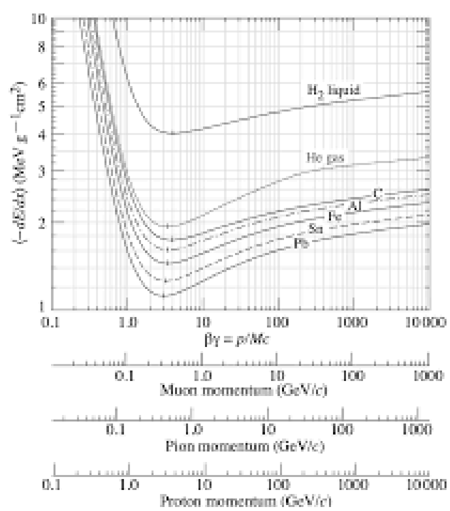
Thus,

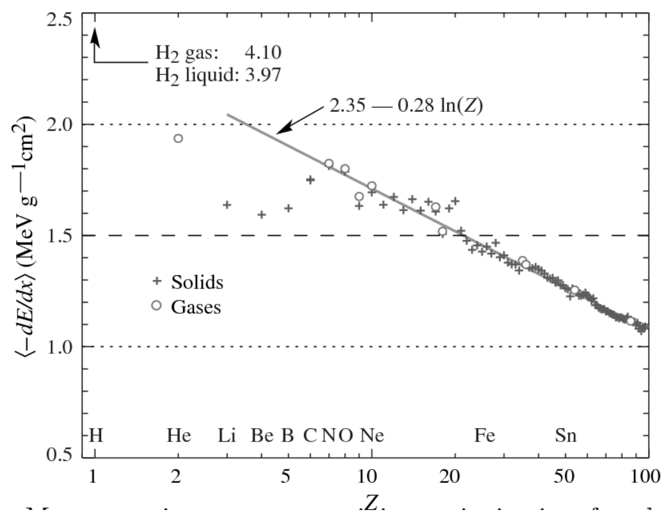
$$\frac{b_{\max}}{b_{\min}} = \frac{e^2}{10\ Z\ 4\pi\epsilon_0}\frac{4\pi\varepsilon_0}{2e^2}2m_e v_\alpha^2 = \frac{m_e v_\alpha^2}{10Z} \equiv \Lambda$$

where $\ln\Lambda$ is called the Coulomb logarithm.

The stopping power (average energy loss per unit length) is then

$$-\frac{dE}{dx} = \frac{16\pi e^4 n_e}{(4\pi\varepsilon_0)^2 m_e v_\alpha^2}\ln\Lambda \tag{1}$$





This must also equal the average energy lost per collision $\Delta K_\alpha = 2m_e v_\alpha^2$ times the expected number of collisions:

$$-\frac{dE}{dx} = \sigma n_e \Delta K_\alpha = 2m_e v_\alpha^2 \sigma n_e$$

where σ is the cross-section for a collision with one electron, and n_e is the number density of electrons.

The cross-section for an alpha-particle-electron collision is, then,

$$\sigma = \frac{8\pi e^4}{(4\pi\varepsilon_0)^2 m_e^2 v_\alpha^4} \ln\Lambda = 2\pi r_e^2 \frac{4}{\beta_\alpha^4} \ln\Lambda$$

where $r_e = \frac{e^2}{4\pi\varepsilon_0 m_e c^2} \approx 2.8$ fm is the classical electron radius, and $\beta_\alpha = v_\alpha/c = \sqrt{2K_\alpha/m_\alpha c^2}$.

1. **What is the cross-section for scattering between a 4-MeV alpha-particle and a nitrogen ($^{14}_{7}$N) electron in barns (1 b = 10^{-28} m^2 = 100 fm^2)?**

2. **What is the cross-section for scattering between a 4-MeV alpha-particle and a lead ($^{208}_{82}$Pb) electron in barns?**

Since the likelihood of energy loss is constant in a given length interval, total energy loss follows an exponential:

$$E(x) = E_0 e^{-x/\ell}$$

where ℓ is the stopping length:

$$\frac{dE}{dx} = -\frac{1}{\ell} E_0 e^{-x/\ell} = -\frac{1}{\ell} E$$
$$\Rightarrow \frac{1}{\ell} = -\frac{1}{E}\frac{dE}{dx} = 4\frac{m_e}{m_\alpha}\sigma n_e$$

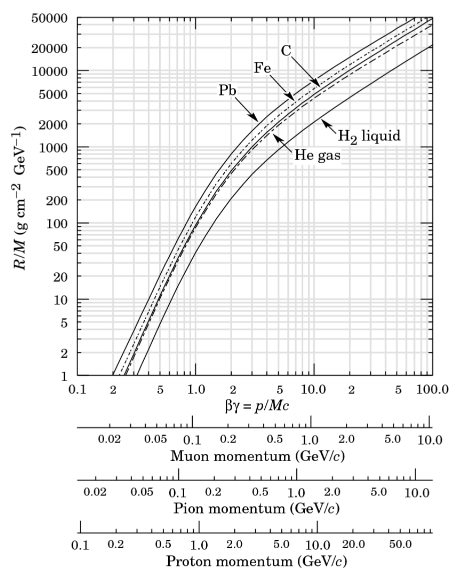
since $-\frac{dE}{dx} = \sigma n_e \Delta K_\alpha$, and $\Delta K_\alpha = 4\frac{m_e}{m_\alpha} K_\alpha$.

3. **What is the stopping length of a 4-MeV alpha-particle in air (air is nearly 80% nitrogen; assume it is only nitrogen)? How far is 10 stopping lengths in air? [Atmospheric nitrogen is a diatomic molecule, $\mathbf{N_2}$; $\boldsymbol{\rho_{N_2} \approx 0.0012}$ g cm$^{\mathbf{-3}}$.]**

4. **What is the stopping length of a 4-MeV alpha-particle in lead? How far is 10 stopping lengths in lead? [$\boldsymbol{\rho_{Pb} = 11.356}$ g cm$^{\mathbf{-3}}$.]**

The distance a particle travels in a material before stopping ($K = 0$) is referred to as the range, $R(K)$:

$$R(K_\alpha) = \int_0^{x(K_\alpha=0)} dx = -\int_0^{K_\alpha} \frac{dE}{dE/dx} \tag{2}$$



The chart shows the range relative to the mass of the projectile as a function of relativistic velocity, $\beta\gamma = p/Mc$. The units of the dependent variable, R/M, are g/cm^2 GeV^{-1}, rather than in some unit of length. The units g/cm^2 refers to the target material, while GeV is the mass of the projectile in natural units. This allows the table to be used for any projectile and any material of any density instead of needing an infinite number of graphs for every possible combination.

To find the range in cm, multiply R/M by the mass of the projectile and divide by the density of the material. As an example, consider a 700-MeV/c charged kaon (a kind of elementary particle called a meson, in this case comprised of a strange quark and an up quark) entering a lead block. The mass of the charged kaon in natural units is about 494 MeV, so, for a 700-MeV/c charged kaon, $\beta\gamma = 700/494 = 1.42$. Locating 1.42 on the plot (note, it's a log-log plot), one finds R/M for lead is about 400 g/cm^2 GeV^{-1}, so $R = 400 \times .494 \approx 198$ g/cm^2. Dividing by the density of lead, $\rho_{pb} = 11.34$ g/cm^3, gives R in cm: $R = 198/11.34 \approx 17$ cm.

5. **What is the range of a 1-GeV/c proton in a bubble chamber that is filled with liquid hydrogen at the boiling point, $\rho_{H_2} = 0.071$ g/cm^3?**

6. **It is claimed that a piece of paper can stop alpha-particles from radioactive decay. These have kinetic energies between 2 and 12 MeV (momenta $\sqrt{2mK}$ between 0.1 and 0.3 GeV/c), with a typical value around 5.5 MeV, or $p \approx 0.2$ GeV/c $\Rightarrow \beta\gamma \approx 0.07$, off the scales of the chart. However, R/M for 20 lb (standard copy) paper ($\rho = 0.89$ g/cm^3, thickness 89 μm) is about 0.0013 g/cm^2 GeV^{-1}. What is the range of the typical radioactive decay alpha-particle in standard copy paper? How far into the paper (fractional distance) will it go before coming to rest? [$m_\alpha \approx 3.7$ GeV.]**

7. **R/M of air ($\rho_{\text{air}} = 0.0012$ g/cm^3) for 5.5-MeV alpha-particles is 0.0012 g/cm^2 GeV^{-1}. What is the range of such alpha-particles in air?**

8. **Alpha-particles make up about 10% of the cosmic ray flux. The momenta of some of these have been measured to be as high as 26 GeV/c. Use the chart to estimate the range in the atmosphere of these high-momentum alpha-particles.**

The Coulomb interaction also mediates the interaction between electron projectiles and target electrons. The ratio of masses in this case is 1, rather than 8000, and electron projectiles are almost always relativistic, since the electron mass is only 0.511 MeV. These interactions result in greater energy changes, larger scattering, and longer stopping lengths than occur in alpha particle interactions. They also must be treated relativistically. The first two effects, large energy changes and scattering, imply that the projectiles undergo substantial acceleration, and accelerated, relativistic, charged objects radiate. This radiation is called bremsstrahlung, from the German for "breaking radiation."

The stopping power, also due to the Coulomb interaction, is similar to that of alpha-particles:

$$-\frac{dE}{dx}|_C = \frac{4\pi e^4 n_e}{(4\pi\varepsilon_0)^2 m_e v_e^2}[\ln\Lambda' + \text{radiative corrections}]$$

where, here, $\Lambda' = \sqrt{\frac{K_e}{2m_e v_e^2}\frac{E_e}{I}}$, where I is the excitation energy, the energy required to raise the atomic electron to a higher energy level.

To this must be added the loss due to bremsstrahlung in order to get the total stopping power.

$$-\left.\frac{dE}{dx}\right|_b \approx \frac{E_e}{m_e}\frac{Z}{1600}$$

The bremsstrahlung contribution becomes significant when the electron is relativistic ($K_e > m_e$) and when the target atomic number is large.

Bremsstrahlung, in the quantum view, is the production of photons with energy equal to the change of the electrons' kinetic energy, $h\nu = \hbar\omega = K_i - K_f$. Each electron can produce many photons in traversing and coming to rest in matter. The energy spectrum of these photons is continuous with a cutoff of the electron energy:

$$K_i = h\nu_{\max} = \frac{hc}{\lambda_{\min}}$$

9. **The Stanford Linear Accelerator (SLAC) was the most powerful linear electron accelerator, reaching energies of 50 GeV. What is the shortest wavelength photon SLAC could produce, and therefore the smallest length that could be probed there? [$hc \approx$ 1.24 GeV-fm.]**

Bremsstrahlung releases electromagnetic radiation within matter. Recall from the previous exercise set the various ways electromagnetic radiation interacts in matter. In almost all cases, an electron is emitted, which becomes another projectile. And so on. This phenomenon is known as cascading or showering.