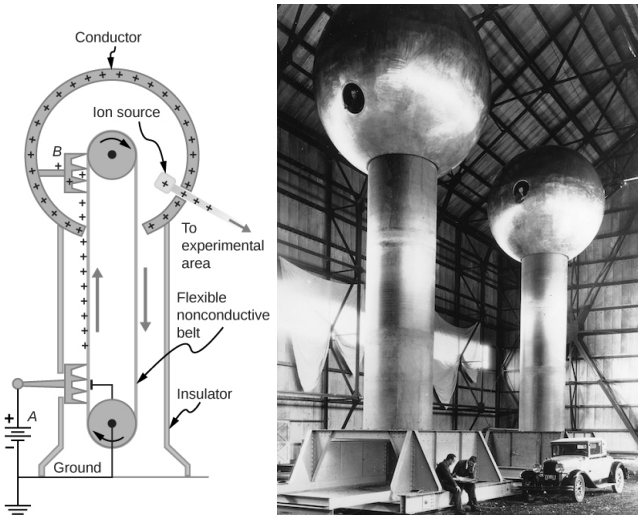
Week 5: Particle Accelerators and Detectors (Undergraduate)

Particle Accelerators

Naturally occurring projectiles are produced in radioactive decays and by cosmic sources, but these have limited intensity, lack energy tunability, and are, by nature, stochastic. Particle accelerators provide high intensity, predictable beams of projectiles at tunable energies. They work by exploiting strong electric and/or magnetic fields.

1. **How much energy does an electron gain when accelerated by a 1-V potential difference?**

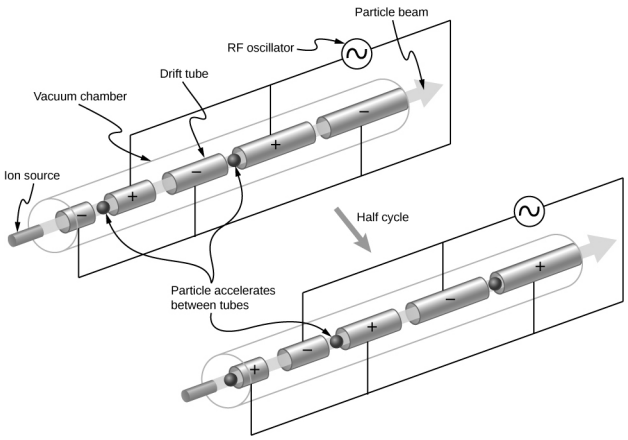
Van de Graaff generators generate large voltages by producing and accumulating large quantities of static charges. The charges are deposited onto a hollow metal sphere and accumulate on its outer surface. The total charge that can be accumulated is limited to potentials of around 16 MV due to the effects of very large electric fields, which polarize and even ionize surrounding materials. Positive ions are then repelled while freed electrons are attracted to the accumulated positive charge, neutralizing them.



Referring to the figure on the left, battery (A) charges a conductor, whose pointed ends are directed at an insulating belt. The high field at the pointed ends cause some of the positive charges to jump to the belt, which transports the charges to the top of the device and another pointed conductor (B), which picks up the charges and conducts them to a large metal sphere. The sphere can accumulate great quantities of charge because each charge moves to the outside of the sphere and distributes evenly there. An ion source inside the sphere produces positive ions, which are accelerated away from the positive sphere with high velocities. Energies produced by a Van de Graaff accelerator are insufficient

to create new particles, but nevertheless aided in the early exploration of nuclear structure.

Higher energies can be attained by repeated acceleration. One of the easiest approaches to do this is a linear accelerator (called a "linac"). Ions are produced at the head of the linac and then accelerated through a series of oppositely charged metal tubes (sometimes called "drift tubes"). The charge, again, resides on the outside. No field exists inside the tube. The voltage difference between each pair of tubes is arranged to push (from behind) and pull (forward) the ions, accelerating them into the next tube. Once inside the tube, the ions travel at their new velocity (drift), while the voltage difference between each pair of tubes is reversed, so as to push the ions out of the tube in which they are drifting and to pull them into the next tube. Each subsequent tube is longer than the preceding one, because the ions are moving faster as they travel down the linac. In short, voltages are alternated such that the ions receive a series of synchronized electric kicks.



Sychronization is trickier as the energy increases, because relativistic effects manifest once the kinetic energy exceeds the mass of the ions. For electrons, this means energies above roughly 1 MeV. Nevertheless, linac technology has advanced to the point of accelerating electrons to 50 GeV ($\beta = 0.99999999994$), and research and development are underway for a 1 TeV electron linac.

In modern linacs, radio frequency (RF) cavities have replaced drift tubes. These cavities set up oscillating electromagnetic fields, which propel the ions forward like surfers riding waves. Such linacs are often the first acceleration stage at accelerator facilities.

2. A linear accelerator, designed to create a proton beam with energy 800 MeV, has 2000 drift tubes. What average voltage difference must be applied in the gap between tubes to achieve the desired energy?

A more compact way of repeatedly accelerating a charged particle is to make it return to the same source of energy, but bending the trajectory requires a field

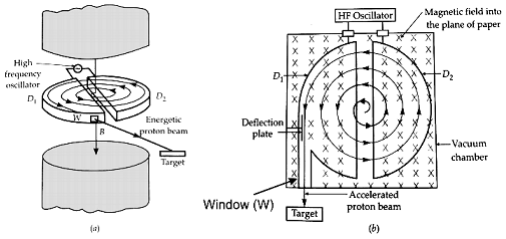
perpendicular to the trajectory and also induces energy losses through radiation emission.

Magnetic fields more efficiently bend the trajectories of charged particles than electric fields, due to the velocity term in the Lorentz force:

$$\vec{F}_{\mathrm{L}} = \frac{d\vec{p}}{dt} = q(\vec{E} + \vec{v} \times \vec{B})$$

where q is the charge, E the electric field, v the velocity, and B the magnetic field strength. Note, too, that $\vec{p} = \gamma(v)m\vec{v}$, to be relativistically correct; $\gamma(v)$ is the Lorentz factor, which depends on the magnitude of the velocity.

The earliest such instrument, called a cyclotron, was developed by E.O. Lawrence. It accelerated positively-charged particles (usually protons, deuterons, or alpha-particles) for nuclear-reaction experiments that produced radioactive isotopes. Cyclotrons employ alternating electric fields and magnetic fields perpendicular to the motion to accelerate particles in a circular, outwardly spiraling path [see Figure].



In a uniform magnetic field, the period of a non-relativistic particle's motion is independent of both the radius of the motion and the particle's kinetic energy.

3. **Show that the period T of a non-relativistic charged particle (charge q and mass m) in a uniform magnetic field of strength B directed perpendicular to the particle's motion is**

$$\boldsymbol{T = \frac{2\pi m}{qB}}$$

and is thus independent of radius of motion and particle velocity. [In order for the period to be in the everyday unit of seconds, all quantities must be in everyday units; recall week 1 exercises.]

Consequently, the period of the alternating voltage source need only be set at the one value. The electric field accelerates particles twice in each orbit.

Particles begin their motion at the center of the cyclotron. The electric field in the gap between two flat, semi-cylindrical metallic containers (dees) accelerates the particles. The dees in turn are enclosed in a metal container, which is situated between–and perpendicular to–the poles of a dipole magnet that provides a uniform magnetic field. In order to minimize scattering and energy loss from collisions with gas molecules, air is evacuated from the large

container. A high-frequency, alternating voltage source is connected to the dees, providing an alternating electric field in the small region between them. No electric field permeates the interiors of the metal dees, but the magnetic field does, inducing the particles to move in an approximately circular, spirling orbit.

Assuming no energy losses in flight, the relativistically correct radius of a charged particle's circular trajectory in everyday dimensions, is

$$r = \frac{\gamma m v}{qB}$$

where m is the particle mass, v the magnitude of the velocity inside the dee, q the charge, B the magnetic field strength. In mixed units, this is

$$r = \frac{p}{0.3B}$$

where r is in meters, $p = \gamma m v$ in GeV/c, and B in Tesla. The (dimensionless) value 0.3 is for a singly-charged particle (that is, the charge has the same magnitude as that of the proton and the electron).

The circular trajectory returns the particle to the electric field gap every half cycle, at which time the electric field is reversed, so that the field always performs work in the direction of motion. With each pass through the gap, the particles gain energy qV, and the radius of curvature of their trajectories increases, and so the particles spiral outward until they exit the instrument.

Since $v/r = \omega = 2\pi\nu = 2\pi/T$, the period of the alternating voltage is

$$T = \frac{2\pi\gamma m}{qB} = \frac{2\pi E}{qB}$$

where E is the total relativistic energy.[1]

In the non-relativistic limit, $\gamma \approx 1$, and the period may remain constant. But when velocities become relativistic, the period or the magnetic field (or both) have to compensate. This is complicated, so, in order to minimize relativistic corrections, projectiles accelerated by cyclotrons tend to be limited to heavier particles, such as protons, deuterons, alpha-particles, and ionized atoms.

Notice, too, that, as the velocity (momentum, energy) increases, the radius increases. The diameter of the magnet thus limits the maximum energy achievable for a specific type of particle. The maximum velocity is

$$v_{\max} = \frac{qBR}{\gamma m}$$

where R is the radius of the dees, and so the energy of the particles at ejection is

$$E = \gamma m = \sqrt{(qBR)^2 + m^2}$$

[1] Again, care must be taken with units for the period to come out in seconds.

In the non-relativistic regime,[2]

$$\begin{aligned} v_{\max} &= \frac{qBR}{m} \\ K &= \frac{1}{2}mv_{\max}^2 = \frac{(qBR)^2}{2m} \end{aligned}$$

4. **The trajectory of a charged particle traces an arc in a uniform magnetic field. What happens to the trajectory if the magnetic field were to double?**

5. **What is the radius of a cyclotron designed to accelerate protons to kinetic energies of 20 MeV with a 2.0 T magnetic field?**

6. **A cyclotron with dees of radius 1.2 m and a magnet which produces a field of 1.5 T accelerates electrons. What is their kinetic energy upon ejection?**

7. **A cyclotron with dees of radius 0.5 m and a magnet which produces a field of 1.8 T accelerates alpha-particles [$m_\alpha = 3.73$ GeV; $q_\alpha = 2e$].**

   (a) **What is the period of the alpha-particles' orbit in the cyclotron?**

   (b) **What is the alpha-particles' kinetic energy when exiting the cyclotron?**

Classic cyclotrons (with static magnetic fields) cannot manage energies beyond non-relativistic limits, because relativistic effects cause the period to increase with orbital radius (effectively, the momentum and energy increase at a faster rate than the velocity because $\gamma = 1/\sqrt{1-\beta^2}$ blows up as $\beta \to 1$). An adaptation, called a synchrocyclotron, attempts to compensate by gradually increasing the magnetic field with orbital radius, but then the limit becomes the size of the magnet.

The limitation imposed by magnet size can be overcome by replacing the single dipole magnet with a series of extended, cylindrical dipole magnets arranged in a circle. This solution has been called a synchrotron: a circular accelerator that accelerates charged particles with alternating voltage in radio frequency (RF) cavities and steers and focuses the charged particle beam by adjusting field strengths in dipole and quadrupole magnets. The RF cavities are synchronized so as to boost the particles' energy as they pass. Steering and focusing high-energy particles requires strong magnetic fields, so superconducting magnets are now often used, reducing heat (power) losses.

It was shown previously that the period of the alternating voltage is given by

[2] Again, to get everyday velocity units, the charge must be in coulombs and the mass must be in kilograms.

$$T = \frac{2\pi\gamma m}{qB}$$

so the frequency of the RF cavities must be some harmonic of

$$\nu = \frac{1}{T} = \frac{qB}{2\pi\gamma m} = \frac{qB}{2\pi E}$$

As the energy (or, equivalently, γ) increases, it would seem that the frequency of the cavity has to decrease. This is not possible, as RF cavities are constructed to very strict frequency specifications. Instead, the magnetic field strength is steadily increased, keeping the radius constant ($r = \frac{p}{0.3B}$), as it must to stay in the machine.

Charged particles deviated from straight-line motion, including moving in a circle, radiate energy. Such radiation, copiously produced at a synchrotron, is called synchrotron radiation. In truth, this radiation can be useful in, for example, materials, biological, and medical research, but, if the objective is to maintain or increase the energy of the circulating particles, the radiated energy has to be replaced.

Experiments use the particles accelerated in the synchrotron as projectiles. The particles are ejected from the ring and aimed at targets for scattering, transmutation, and particle creation reactions.

8. **The discovery of the antiproton: A single-magnet (soft-focusing) synchrotron, called Bevatron (beva was once used instead of giga–think "billion") was built by University of California, Berkeley, to see if the antiproton (like the antielectron–positron–which had already been discovered) exists. The idea was to accelerate protons and smash them into a proton (hydrogen) target at energies in excess of the threshold for creating an antiproton. By fundamental theoretical considerations, and in analogy with the positron, the expectation was that, if it exists, the antiproton's mass should be the same as the proton's. To conserve charge and baryon number (antiparticles have charge opposite their particle partner; the proton is a baryon, and the number of baryons had always been conserved in nuclear and particle reactions), the minimum energy reaction would be**

$$\boldsymbol{p + p \rightarrow p + p + p + \bar{p}}$$

where $\bar{p}$ is the antiproton. What minimum kinetic energy must the Bevatron supply? [The four-vector-squared is invariant, $\boldsymbol{P^2 = M^2 = E^2 - |\vec{p}|^2}$, in any inertial reference frame; the energy of a particle in motion is $\boldsymbol{E = K + m}$, while the energy of a motionless particle is its mass $\boldsymbol{E = m}$; the lab frame is easiest to work in for the initial state, while the CM frame, where the 4 final state particles are at rest at threshold, is easiest for the final

state; calculate the invariant four-vector-square for each state, set them equal, and solve for K.]

Colliding beam machines–or colliders–accelerate two counter-rotating beams to the desired energy, and then stores them at constant energy before steering and focusing them to collide at the center of a particle detector.

9. Colliders are classified as symmetric or asymmetric, depending on characteristics of the beams and beam particles. What is meant by a symmetric collider? What would be different in an asymmetric collider?

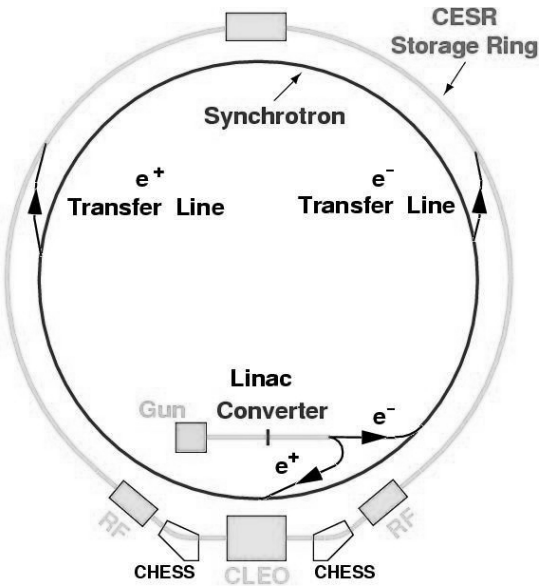
The center-of-mass energy (W) is the total (relativistic) energy of particles colliding in the center-of-mass. It is the total energy available to create new particles in a colliding machine or to probe unexplored shorter distances.

$$\begin{aligned} W^2 = M^2 &= (E_1 + E_2)^2 - (\vec{p}_1 + \vec{p}_2)^2 \\ &= (E_1^2 - |\vec{p}_1|^2) + (E_2^2 - |\vec{p}_2|^2) + 2(E_1 E_2 - \vec{p}_1 \cdot \vec{p}_2) \\ &= m_1^2 + m_2^2 + 2(E_1 E_2 + |\vec{p}_1||\vec{p}_2|) \end{aligned}$$

where E_1 and E_2 are the total energies of the beam particles, $\vec{p}_1$ and $\vec{p}_2$ their momenta, and m_1 and m_2 their masses. Note that, in the center-of-mass, $\vec{p}_1 = -\vec{p}_2$.

10. The result of problem 8 might have been surprising. Initially there are two protons and finally there are four (counting the antiproton as a proton), so why isn't an additional $2m$ or $3m$ of energy sufficient. The problem is that momentum must be conserved, and colliding a beam into a fixed target "wastes" a significant amount of kinetic energy in order to do so. If, instead, the two initial state protons, each with energy E are directed into a head-on collision (that is, the laboratory and CM frames are now the same), what would the kinetic energy of each proton have to be to create an antiproton in the same reaction?

To date, colliding beam machines have been built to accelerate electron-positron, proton-antiproton, proton-proton, and ion-ion beams. Designs for a muon-antimuon collider have been proposed. The now decommissioned 768-meter circumference Cornell Electron Storage Ring (CESR), located on 12 meters underground on the Cornell University campus, in Ithaca, New York, is a nice example of a colliding beam machine.



Free electrons (e^-) were accelerated to 150 MeV in a linear accelerator. A thin target could be inserted into the linac for the creation of positrons (e^+). Particle bunches were then injected in turn into the inner synchrotron accelerator, where RF cavities accelerated them to between 4.5 and 6 GeV, depending on the intended energy regime being investigated. When each beam was at its intended energy, it was transferred into an outer "storage" ring at the same energy. The electrons and positrons were injected into the storage ring in opposite directions, so that two counter-rotating beams traveled through the same evacuated beam pipe. The two beams were kept apart until collisions were desired. The electrons and positrons circled the ring in bunches 390,000 times every second.

When they collide, an electron and a positron annihilate one another. At low energies, annihilation produces two real photons, whose momentum and energy conserve these two quantities. At high energies, the annihilation produces a virtual photon whose energy is that of the center-of-mass energy, but it exists for too short a time to be detected. The virtual photon transforms into either a lepton pair (electron-position, muon-antimuon, tau-antitau, neutrino-antineutrino) or a quark-antiquark pair. The quarks in turn pair together as mesons (quark-antiquark objects).

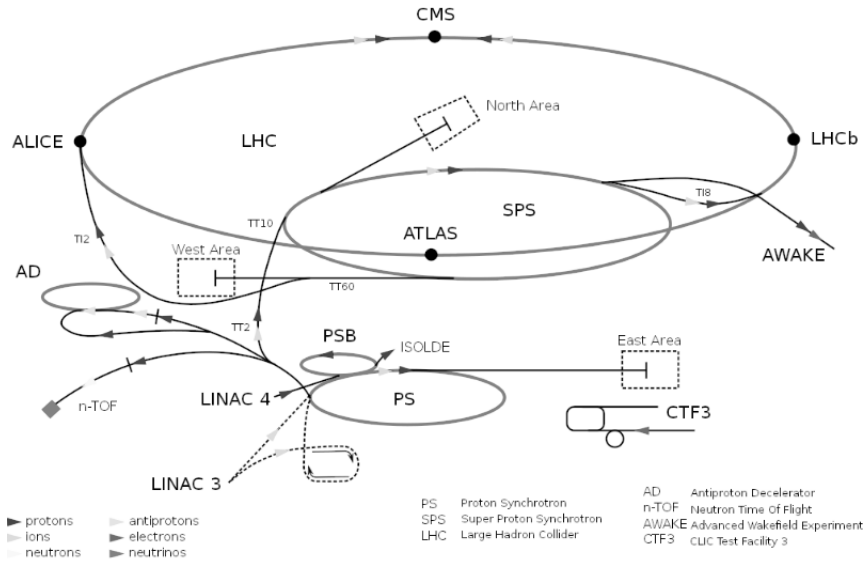
At CESR, the center-of mass momentum was zero, so the pairs flew off in opposite directions. Given CESR's beam energies, when a tau lepton (the heaviest lepton) is created, or a quark-antiquark pair forms a heavy meson (such as charm or bottom mesons), these remained nearly at rest in the laboratory.

The heaviest mesons formed in these sorts of annihilations are resonance states: they are created when the collision energy is at the energy of the meson. Resonances are unstable, transforming in roughly 10^{-21} s into lighter, more stable particles, including protons, electrons, and photons. These lighter particles

are detected and their characteristics are measured, The original interaction is surmised by analyzing these characteristics.

11. CESR specialized in the physics of the Υ (upsilon) resonance (made of a $b-\bar{b}$ pair), $m_\Upsilon = 10.58$ GeV. At what energy did CESR run when creating these mesons? [Note that $E_{\text{beam}} \gg m_e$.]

Particle physics advances by searching for ever heavier unstable objects and investigating ever shorter distance scales. Both endeavors require increasingly powerful accelerators. The Large Hadron Collider (LHC), near Geneva, Switzerland, is currently the world's most powerful accelerator. It is part of the CERN accelerator complex. The protons that it collides have energies around 7 TeV (10^{12} eV). They are supplied by hydrogen gas at room temperature ($K \approx 0.04$ eV). The separated ions are first accelerated in a linac (Linac 4) to 50 MeV before injection into a synchrotron (Proton Synchrotron Booster) that increases the energy to 1.4 GeV. A larger synchrotron (Proton Synchrotron) raises the energy to 25 Gev. The largest synchroton in the complex (Super Proton Synchrotron) then increases the proton's energies to 450 GeV before injecting them in counter-circulating bunches into the LHC for final acceleration and collision.



Accelerator design involves optimizing (maximizing) the energy while (minimizing) the size. One of the most severe limiting factors is the cost of replacing the energy lost to synchroton radiation (from the acceleration of charged particles). Synchrotron radiation goes as

$$\Delta E \propto \frac{\gamma^4}{R} = \left(\frac{E}{m}\right)^4 \frac{1}{R}$$

But, as we have seen, the energy is related to the radius and magnetic field strength:

$$E \approx p = 0.3BR$$

so another limiting factor is the strength of the magnets, which have maximum values of 1 T if standard (warm) iron-core electromagnets or 5T if superconducting. Note that this is independent of projectile particle type.

12. **What is the mimimum radius for a 100 TeV (center-of-mass energy) collider built with the most powerful warm magnets? With the most powerful superconducting magnets?**

13. **At the minimum radius for a 100 TeV (CM energy) collider with superconducting magnets, how much more energy do electrons lose to synchroton radiation than do protons?**

Particle Detectors

An experiment in nuclear or particle physics aims to clarify one or more subatomic reactions. The equipment used in such investigations is collectively known as a detector. Nuclear and particle physics detectors are comprised of subdetectors, each designed to measure a different aspect of the reactions under investigation. In practice, only stable or especially long-lived particles are detectable, and it is their characteristics and behaviors that are measured. Thus, the job of one or more subdetectors might be to define the trajectories and timing of the particles, or to measure their energies or momenta, or to identify what they are. Any one subdetector might perform one or more of these jobs on photons, another on electrons, and so on. The information from all subdectors is assembled on an individual reaction (event-by-event) basis, with the intent of reconstructing what happened during the reaction.

Reconstruction of a subatomic reaction implies satisfying the relativistic momentum-energy equation

$$m^2 = E^2 - |\vec{p}|^2$$

not just for each constituent of the reaction individually, but for all of them simultaneously

$$M^2 = (\sum E)^2 - |\sum \vec{p}|^2$$

Since each m and M is invariant, and since each subatomic object is uniquely identified by its mass, satisfying these equations tells us what the parent and all daughters of the reaction were. The existence of these relationships implies that it is necessary to measure only two of the three characteristics–mass, energy, and momentum. The third is determined by the relationships. In fact, it is not always necessary to directly measure even two of them. For example, m can

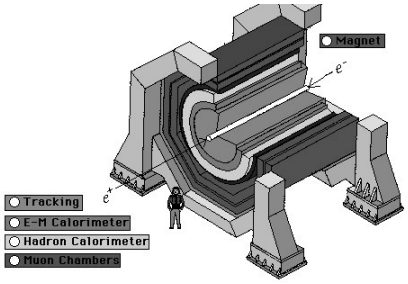
also be determined by measuring the momentum and the velocity separately, or by measuring the energy and the velocity separately, since,

$$\begin{aligned} |\vec{p}| &= \beta\gamma m \\ E &= \gamma m \end{aligned}$$

In short, reconstructing a subatomic reaction requires determining some or all of the following:

- where the constituents were and when they were there (tracking and timing);
- the momenta, energies, and/or velocities of the constituents (tracking and calorimetry); and
- the identities of reaction constituents (particle identification); and,
- (sometimes) the constituents' spins, angular momenta, parities, and other quantum numbers.

Rarely can a single instrument make all these measurements. Rather, combinations of subdetectors are integrated into a deteector system.



Typically, tracking instruments, often embedded in a magnetic field, are installed closest to the reaction, followed in turn by those with precise timing and energy measurment (calorimeters). Interspersed and usually following are instruments for indentifying the various constituents.

The quality of an experiment, usually quantified as a resolution or relative uncertainty, is limited by time, space, and energy resolution; detection efficiency; and particle misidentification probability. Momentum resolution is limited by space resolution and magnetic field strength. Velocity resolution is limited, depending on measurement method, by time resolution (time-of-flight), energy resolution (dE/dx), and space resolution (Cerenkov angle measurement).

Because charged particles ionize the material they pass through, they leave a potentially readable record of their passage.

Emulsion films[3] were first created in the 1940s, but are still used in neutrino experiments and medicine. Ionizing particles passing through the emulsion break up silver halide (AgBr and/or AgCl) molecules suspended in a geletin coated (usually) on a glass substrate. The resulting silver ions efficiently trap electrons shared by four or more of the ions to form a cluster, a process known as nuclearization. These clusters form what is known as a latent image center, whose diameter is typically about 0.5 μm. Developing reduces the clusters to metallic silver and fixes them to the substrate, while rinsing away remaining geletin and silver halide molecules. The fixed silver clusters appear as black dots when viewed with a microscope.

Good reasons for using emulsions include their excellent spatial resolution and there ability to provide both velocity (dE/dx) and momentum (amount of multiple scattering) measurements simultaneously. Good reasons for not using emulsions include their limited volume, their lack of triggerability, and the destructive nature of their processing.

14. A particle involved in a reaction is detected in an emulsion, from which its momentum and kinetic energy are determined to be $|\vec{p}| = 330$ MeV and $K = 215$ MeV.

(a) Show that a particle's mass is given, in terms of its momentum $|\vec{p}|$ and kinetic energy K, by

$$m = \frac{|\vec{p}|^2 - K^2}{2K}$$

(b) What is the particle's mass, m?

(c) What is the particle's velocity, β?

(d) What is the particle's identity?

A charged particle passing through a gas can ionize the atoms or molecules of the gas. The gas gains electrical conductivity due to the presence of these ions and electrons, which can be collected to determine where and when the charged particle passed. Devices capable of doing this consist of a container to hold the gas and two electrodes across which is a potential difference.

Proportional counters consist of gas-filled metal or glass cylindrical tubes that are kept at a negative potential (cathode). Strung down the center of the tubes are single fine wires set to a positive potential (anode). Ionization electrons are accelerated toward these wires.

If the potential difference is V_0, the tube radius is r_t (typically around 1 cm), and the wire radius is r_w (typically, 20 - 100 μm), then the internal electric field is

[3]Such emulsions are similar to, but distinct from, optical photographic film. The geletin layer is about 50 times thicker, and the silver halide molecules are more than 50% denser, than the suspension of optical photographic film. The silver crystal grains are also on average about 1/10 the size, and are more uniform in size and distribution than in photographic film.

$$E(r) = \frac{V_0}{r \ln(r_t/r_w)}$$

When, due to an ionizing interaction, an electron is freed a distance r_a from the center of the wire whose radius is r_b and drifts to the wire, it gains kinetic energy

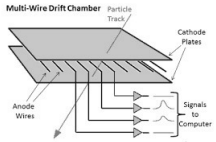
$$K = e \int_{r_a}^{r_b} E(r) dr$$

Once K exceeds the gas's ionization energy, additional electrons are released, which in turn ionize atoms closer to the wire. This avalanche of electrons is quantified as the amplification factor or gain–the number of secondary electrons reaching the wire per initial ion pair–which is characteristic of the gas and independent of the number of initial ion pairs. Thus, the number of electrons reaching the wire is proportional to the initial ionization, hence, the name of the device: proportional counter.

A typical amplification factor/gain is around 10^5. Also typical is the approximately 100 electrons per cm liberated by a traversing charged particle.

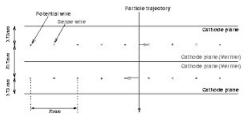
There's an upper limit to V_0 before a breakdown (spark), which can damage the fine wire, will occur. But this can be controlled and exploited under certain conditions, in particular by mixing in a quenching gas–usually a complex organic molecule–that readily captures electrons. With the right mixture and pressure, the discharge can propagate along the entire wire length. This is how a Geiger-Muller counter works. If, instead, the very high voltage is pulsed, the original avalanche can be localized. Gas tubes used in this way are known as streamer chambers.

Modern proportional counters (called multiwire proportional chambers, MWPC), as developed by Georges Charpak (Nobel Prize 1992), obtain spatial information by having many fine parallel wires in a plane as one set of electrodes, between planar steel meshes serving as the other set of electrodes. The applied voltage is typically 2000 V, with wires of diameter of order 50 μm and separations of 0.1 to 1 cm. The gas is usually an argon/alcohol mixture, at atmospheric pressure. Each wire has its own transistor amplifier, and acts like a cylindrical proportional chamber. Amplification factors are of order 5000, sufficient for single minimum-ionizing particles to be detected with 100% efficiency. Two counters, with perpendicular wires, can give the spatial position of a particle to about 5 mm.



Good spatial and time resolutions with a MWPC requires an enormous number of wires and amplifiers. Eqivalent resolutions at lower cost can be achieved

with a device called a drift chamber, in which the freed electrons are channeled by electric fields to drift from the primary ionization through a low-field region of roughly 10 cm to the high-field avalanche region near the anode wire. The time difference between a fast trigger and the time of the pulse on a wire gives the distance from the wire of the primary ionization, $r = vt$, where $v \approx 50$ μm/ns in a typical argon-alcohol (butane or ethane) mixture. The typical drift time of around 25 μs means that drift chambers are suitable only for relatively low incident particle rates.



Tracking is also performed with silicon detectors. Though solid, their principle of operation is same as the gas detectors, but with electron-hole pairs instead of ion pairs produced by ionization.

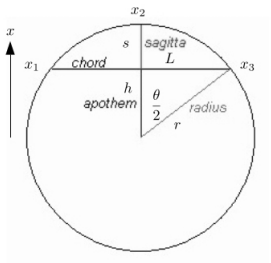
The electron density in silicon is a few thousand times larger than it is in gases, and the ionization energy is a factor of 10 smaller ($\sim$3 eV vs $\sim$30 eV), so large signals are available without the need for gas gains in high voltage. The detectors are thin and the drift velocities of both electrons and holes are high, so the detectors are both fast and have excellent time resolution. Narrow electrode strips and small detector pixels provide $\sim$5 μm position resolution. These are very expensive, and so resaonable only for small instruments, usually around the interaction region, providing primary vertex information.

15. Explain in your own words how tracks are created in particle detectors.

Embedding tracking detectors in magnetic fields provides a means for measuring the momenta, p, of charge particles, since

$$p_\perp = rzeB$$

for p in GeV; B, the strength of the magnetic field, in tesla; r, the radius of curvature, in meters; and z, the charge of the traversing particle, in number of unit (proton) charges. In these hybrid units, $e = 0.3$, the magnitude of a unit (proton) charge.



The radius of curvature, and therefore the momentum, can be expressed in terms of the sagitta of the trajectory, if three positions have been measured along the trajectory,

$$s = x_2 - \frac{x_1 + x_3}{2}$$

Assuming $L \ll r$,

$$L = r\frac{\theta}{2} \Rightarrow \theta = \frac{2L}{r}$$

where L is half the chord of the trajectory and is known by the locations of the measuring devices.

Because $r = s + h$ and $h = r\cos\frac{\theta}{2}$,

$$s = r(1 - \cos\frac{\theta}{2}) = 2r\sin^2\frac{\theta}{4} \approx \frac{1}{8}r\theta^2$$

since $\sin^2\alpha = \frac{1}{2}(1 - \cos\frac{\alpha}{2})$, and $\sin\alpha \approx \alpha$ for $\alpha \approx 0$. Then,

$$s \approx \frac{L^2}{2r} = \frac{zeBL^2}{2p_\perp}$$
$$\Rightarrow p_\perp = \frac{zeBL^2}{2s}$$

since $r = \frac{p_\perp}{zeB}$.

If all individual position measurements are known to the same resolution σ_x, then

$$\sigma_s = \sqrt{\frac{3}{2}}\sigma_x$$

so that

$$\left(\frac{\delta p_\perp}{p_\perp}\right)_{\text{spa}} = \frac{\sigma_s}{s} = \sqrt{\frac{3}{2}}\sigma_x\frac{2p_\perp}{zeBL^2} = \sqrt{6}\sigma_x\frac{p_\perp}{zeBL^2}$$

With more measurements, the contribution of one measurement to the overall uncertainty decreases. As the number N of measurements becomes large

$$\left(\frac{\delta p_\perp}{p_\perp}\right)_{\text{spa}} = \sqrt{\frac{45}{N+4}}\sigma_x\frac{p_\perp}{zeBL^2} \tag{1}$$

So the position resolution contribution to the momentum resolutions becomes worse with momentum but improves with B and L as $1/BL^2$.

Multiple scattering also contributes to the momentum uncertainty independent of the position resolution. In fact, this dominates the uncertainty at low momentum, but becomes less important at higher momentum.

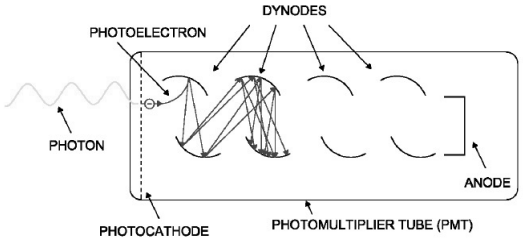
16. A singly charged particle in a 2.0-T magnetic field is bent in a circle of radius 75 cm. What is the momentum of the particle?

17. A proton passes through a magnetic field and traverses a circular path of radius 50 cm. The magnetic field strength is 1.5 T. What is the total energy of the proton?

Nuclear and particle reaction experiments involve one or more projectiles and one or more final state particles. In order to obtain sufficient statistics, the experiments attempt to repeat the reaction as many times as necessary and feasible. The event rate is therefore usually quite high. In addition, extraneous processes are likely to introduce additional particles. To allow experimenters to sort out what is associated with a particular reaction, certain detector elements are designed for excellent timing.

Scintillation counters are among the most frequently used instruments for this purpose in nuclear and particle physics. Modern scintillation materials include plastics such as polystyrene, organic liquids (for example, toluene); inorganic crystals (such as sodium or cesium iodide), and organic crystals (like anthracene). Charge particle-induced ionization results in emission of fluorescent light over a broad range of wavelengths. In organic scintillators, for example, the emission is principally in the short (UV) wavelength region. A wavelength-shifting dye is usually mixed into the scintillator material. The UV emissions of the scintillator materal excite the dye, which reemits in the visible region. The decay times of fast scintillator (the time between excitation and reemission) is typically of order 1 ns. The light output of inorganic crystals, such as NaI(Tl), is essentially proportional to the ionization energy loss. These are therefore used for calorimetry. This is not the case with organic scintillators, and these are ordinarily the type used for timing.

Scintillators are usually wrapped in a layer of reflecting material, such as aluminum foil. One or both ends of the slabs are left open. The point of the wrapping is to reflect as much light as possible to these exposed ends, to which is attached a photosensitive detector, such as a photomultiplier tube, or a light guide to which is attached a photosensitive detector.



At the end of a photomultiplier tube attached to the face of a scintillator or light guide is a photocathode, which is coated with alkali metals, so that electrons are readily emitted (due to the photoelectric effect). The emission

efficiency, known as the quantum efficiency, depends on the wavelength of the light that strikes it. Maximum photocathode quantum efficiency

$$QE(\lambda) = \frac{N_e}{N_{\gamma(\lambda)}}$$

is typically around 25% for $\lambda = 400$ nm light.

A minimum ionizing particle is a particle whose mean energy loss rate through matter is close to the minimum of the dE/dx distribution. In general, relativistic particles with $\beta\gamma \approx 3$ are minimum ionizing. Such particles lose about 1.5 MeV of energy to ionization traversing 1 cm of plastic scintillator. Such a traversal liberates on average 10^4 photons, each with a mean energy $hv = 3$ eV, meaning that about 2% of the energy loss results in fluorescence. Assume 10% of the light makes it to the photocathode, whose quantum efficiency over the full spectrum is 10%, then approximately 2000 photoelectrons will be emitted, sufficient to ensure nearly 100% efficiency.

For each photoelectron ejected from the cathode, the successive secondary-emission electrodes (dynodes), which emit on average 4 secondary electrons for each incident electron, ultimately generate some 10^8 electrons at the anode. The transit time from cathode to anode is about 50 ± 1 ns, where the uncertainty is known as the jitter. The output of the anode is a voltage pulse.

The width of the output pulse, referred to as the response time, depends on the time required for the light to make its way to the photocathode and the fluorescence decay time–between 3 and 30 ns for organic materials and around 250 ns for NaI–as well as the photomultiplier jitter. The minimum width, then, is around 10 ns for plastic scintillator.

The great advantages of the scintillation counter are that it is robust, simple, and efficient, giving large, sharp output pulses. Its spatial resolution is poor, since the pulse is not related in an obvious way to the location of the charged particle's passage through the counter. Thus, if spatial information is required, it is necessary to use an array of very small counters with photomultipliers at each end. Coincident signals from each of them reduce random noise that characterizes all photomultiplier tubes, while the time difference between the two signals allows an estimation of the traversal point in a counter. The rate of accidental coincidences is small because of the good intrinsic time resolution.

Rather than exciting the atoms of the material on its way through, a particle traversing some other sorts of material can lose its kinetic energy to a shower cascade, the outcome of multiple ionization or excitation processes. Detectors comprised of these materials can therefore provide not just timing, but also energy and position information. The fractional energy resolution varies as $1/\sqrt{E}$, so, at high energy (> 10 GeV), the resolution approaches that of the momentum resolution of trackers in magnetic fields, but the time resolution is better than that of gas tracking detectors by a factor of 10 (100 ns vs 1 μs).

The electromagnetic cascading of photons and electrons through radiation energy loss (bremsstrahlung and pair production) in materials allows their total energy to be measured along the path of the shower. Coulomb scattering spreads

the shower laterally, the extent of which depends on the radiation length, X_0,[4] a characteristic of the material, and the angular deflection per radiation length at the critical energy, E_c, at which shower development ceases. This is expressed in terms of the Molière radius

$$R_m = 21 \left(\frac{X_0}{E_c} \right)$$

which is, by definition, the radius of a cylinder containing on average 90% of the radiated energy. Two Molière radii contain 95% of the energy. The smaller a material's Molière radius, the greater resolution of the shower position, and the less likely it is that multiple showers will overlap (shower separation will be improved).

High Z materials with short radiation lengths are preferred for electromagnetic calorimeters. Typical relative energy resolutions are given by

$$\frac{\Delta E}{E} = \frac{0.05}{\sqrt{E}}$$

where E is measured in GeV. This results from the facts that the number of particles at shower maximum $N \propto E$, and this number fluctuates as $\sqrt{N}$.

Similarly, a hadron traversing matter can produce a shower if it collides inelastically with a nucleus. The longitudinal extent of such showers depend on the nuclear interaction length λ_I.

While the energy of electrons and photons ultimately leads to ionization, so that ionized electrons can be collected as a measure of energy, hadrons lose roughly 30% of their energy through nuclear fission, excitation, and evaporation (emission of individual nuclei from a nucleus), none of which will produce observable signals. This can be compensated somewhat through the use of materials that release nucleons with sufficient kinetic energy to ionize atoms in the material, but the typical energy resolution of a hadron calorimeter is an order of magnitude worse than the energy resolution of an EM calorimeter:

$$\frac{\Delta E}{E} = \frac{0.5}{\sqrt{E}}$$

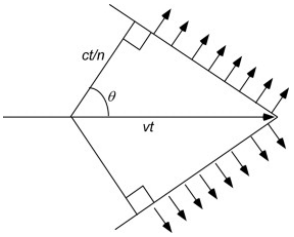
The nuclear interaction length tends to increase with Z, while the radiation length tends to decrease, so hadronic showers tend to be much larger, both longitudinally and transversely, than electromagnetic calorimeters.

Particle speed is sometimes measured directly by time-of-flight instruments, two fast, high-precision detectors, such as scintillator counters, separated by a known distance L. The difference between the traversal times at these detectors gives the speed, $\beta = L/\Delta t$. Then, a separate momentum measurement gives the particle identity:

[4] For a single-element material, $X_0 = 1433 \text{ g cm}^{-2} \frac{A}{Z(Z+1)(11.219 - \ln Z)}$, and $1/X_0$ is the absorption coefficient of the negative exponential giving the absorption law.

$$m = \frac{p}{\beta\gamma} = p\sqrt{\frac{1}{\beta^2} - 1} = p\sqrt{\frac{(\Delta t)^2}{L^2} - 1} \tag{2}$$

Instruments that can measure ionization density ($\frac{dE}{dx} \propto \frac{1}{\beta^2}\ln(\beta^2\gamma^2)$) and Cerenkov detectors ($\beta = \frac{1}{n\cos\theta}$) are other tools to determine speed. Ionization density was discussed in the previous exercise set. Cerenkov counters are based on the electromagnetic wave analogue of a supersonic boom. When a particle's velocity exceeds that of the speed of light in a medium, $v > \frac{c}{n}$, where n is the medium's index of refraction, a shock wave is produced, and a linear wave-front propagates through the medium [see Figure].



The angle of the wave-front relative to the direction of the particle is related to the particle's speed by

$$\cos\theta = \frac{1}{\beta n}$$

Particle identification is also made possible by the fact that the different final state particles interact in different ways in different materials.

Traversing electrons leave ionization trails in low-density matter and shower electromagnetically while their energy is absorbed in dense matter. The showers begin forming rather deeply into the material and are relatively short and well-collimated.

Traversing photons do not leave ionization trails in low-density matter, but do shower electromagnetically while their energies are absorbed in dense matter. The showers begin forming near the front surface of the material and are relatively short and well-collimated.

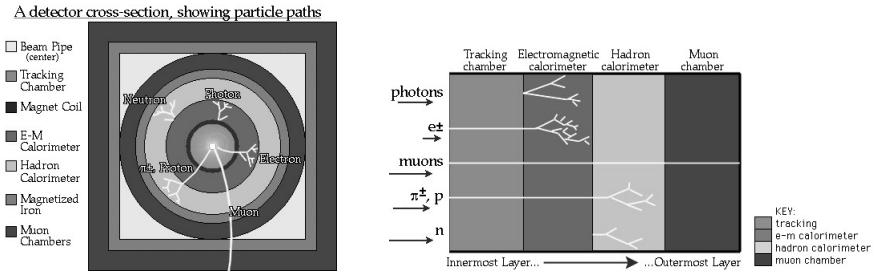
Sufficiently long-lived charged hadrons, principally protons, pions, and kaons, leave ionization trails in low-density matter and shower hadronically while their energies are absorbed in dense matter. The showers may begin forming at any depth in the material and are long and diffuse.

Sufficiently long-lived neutral hadrons, principally neutrons and kaons, do not leave ionization trails in low-density matter, but do shower hadronically while their energies are absorbed in dense matter. The showers may begin forming at any depth in the material and are long and diffuse.

Muons, which are charged, leave ionization trails in both low- and high-density matter, but very rarely shower, so their energies are rarely absorbed.

Neutrinos, which are neutral, do not ionize material, leaving no trails of there passage through it. Their presence must be inferred by missing energy or momentum imbalance.

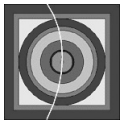
The identity of all other particles is inferred by reconstructing the energy and momentum of the particles just listed. Such reconstruction can result in invariant masses and displaced vertices.



18. Identify the particles that left these track:

(a)

(b)



(c)

(d)